GAMMA SPECTRAL CALCULATIONS BY THE MOMENTS METHOD

A Thesis

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by

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ABSTRACT

The semi-numerical technique 'Moments Method' proposed by Spencer and Fano and later expanded by Goldstein and Wilkins has been carried further with P-7 approximation. A computer program has been developed to calculate the moments for the plane isotropic source in infinite homogeneous medium. Differential angular energy spectra, $4\pi r^2 e^{\mu_o r} I_o S$, and dose build-up factor (B_r) have been calculated for water at 1 and 3 Mev, and for concrete, aluminum, and iron at 1 Mev at the depth of penetration ($\mu_o r$) 1, 2, 4, 7, 10, 15 and 20 for the point isotropic source. The results at the relaxation length 10, 15 and 20 are significantly different from those reported previously. The absorption coefficient of concrete, treated as an homogeneous mixture, was calculated in TU/electron.

CHAPTER 1

INTRODUCTION

During the last two decades, nuclear reactors and other intense sources of gamma rays have found tremendous use in almost every aspect of scientific research and applications. Such use calls for reasonably accurate calculations of the shielding or gamma ray blocking provided by various media. Semi-empirical methods as well as Monte Carlo methods have been used in the shielding calculations. They are either too machine time consuming or only partially true for the gamma ray interactions. L. V. Spencer and U. Fano, et al [12] of National Bureau of Standard proposed a semi-numerical technique for solving the Boltzmann equation with certain simple geometries. Considerable effort has been devoted to developing methods of calculating the deep penetrations of photons in media of common shielding material. Time independent Boltzmann's equations, in the majority of the cases, represent a unique mathematical description of gamma ray interaction in material media and 'Moments Method' is one of the techniques, semi-numerical in nature, used in the computation of gamma ray shielding-design problems.

Intense gamma sources pose a problem in the nature of the attenuation because of the geometry of the media under consideration.

Measurement of the attenuated beam becomes a difficult experimental task because of the geometry of the system. Precisely, if the transverse dimensions of the material media through which the photons are penetrating is of the same order of magnitude as the longitudinal dimensions, the medium is said to have "bad geometry"[3]. A nuclear reactor and its surrounding shield is an example of "bad geometry" configuration where the simple attenuation factor ($e^{-\mu x}$), as expressed in Equation (1)

$$I = I_o e^{-\mu x} \qquad (1)$$

I is the observed intensity at a distance x in a medium having attenuation coefficient, μ, and I_o is the incident gamma intensity, no longer holds good. Scattering of gamma rays is staggering in "bad geometry" configuration and must be considered in shielding design problems.

'Moments Method' is a good computational procedure for shield design problems and like any other method, has its limitations. The computations are limited to

1) infinite homogeneous media
2) simple source geometries, such as point isotropic or plane isotropic, plane monodirectional, or infinite uniform plane sources
3) monoenergetic sources

4) physical behavior of the penetration of the gamma-ray can be well represented by the time independent Boltzmann's gamma transport equation.

The present work is an extension in the computational procedure for point isotropic sources where eight moments have been calculated and used during the reconstruction of the differential angular energy $[4\pi r^2 I_o S(r, \lambda) e^{\mu_o r}]$ flux. Results at relaxation lengths ($\mu_o r$), such as, 1, 2, 4, and 7 show very close agreement as reported[3] but significant deviations have been observed at the relaxation lengths, 10, 15 and 20. Differential angular energy spectra when plotted against energy showed a different behavior than reported at relaxation length 10, 15 and 20. No convergence analyses have been made in this work; more stress was given to the machine computational procedure - the novelty being, use of more functions subprograms and fewer subroutines, thereby economizing storage space in the computer and saving the computation time.

Since no data were available in the literature on moments computed for plane isotropic or point isotropic sources, accuracy of the computed moments could not be checked. All of these computations are believed to be accurate to seven significant digits and some were cross-checked by hand calculations. Numerical errors during the integration loop, and interpolation of data on energy and photon wave length for calculating absorption coefficients of the medium in Thompson unit/electron are inherent problems associated

with the computation of the moments. The approximations associated with biorthonormal polynomial functions during the final reconstruction of differential angular energy flux adds further to the error. It is extremely important to have accurate absorption coefficient data for air during the computation of dose (Mev/cm^3sec) and dose build-up factor. Absorption coefficient of air is quoted[1] in units of cm^{-1}. In the calculation of total dose, consisting of the scattered and unscattered gamma rays, it requires ratio of the absorption coefficients. It is not critical at all in which unit the absorption coefficient of air is used since the ratio becomes dimensionless. It is imperative, however, to decide the number of moments used in the calculation, since the magnitude of the differential angular energy flux is dependent on the number of moments used for the reconstruction of the angular energy flux. $\sum_{n=0}^{N} B(2n, 0)$ is required in the computation of flux. Choice of N is critical for convergence. The extensive work as reported in NY03075 for plane isotropic moments calculations uses six moments (N varies from 0 to 5), the present work extends this to eight moments (N varying from 0 to 7). Consequently, seventh order of biorthonormal functions (Lagurre functions) had to be used in the calculation. Consequently, the differential energy flux has different values than reported[3].

Total gamma dose rate (Mev/cm^3sec) has been calculated for water at 1 Mev. Dose buildup factor has been computed for materials

such as concrete, aluminum, water and iron at 1.0 Mev. Calculations have been extended for water medium at 3 Mev. It is worth mentioning that moments are neither a monotonic decreasing nor a monotonic increasing function of photon wave length (or gamma energy) - their magnitude could also be negative. This strange behavior of moments could be reasoned as follows. During the computation of moments it is necessary to obtain the product of the Kline-Nishina kernal and the Legendre polynomial $P_\ell(1 + \lambda_0 - \lambda)$, for the particular choice of the order (ℓ). Since the argument of the Legendre function, $1 + \lambda_0 - \lambda$, could be either positive or negative, therefore the moment could be either positive or negative. It is difficult to predict the magnitude of moments, except that it behaves as a "wavy" function, close in nature to that of the Legendre polynomial $P_1(1 + \lambda_0 - \lambda)$.

CHAPTER 2

INTERACTIONS OF GAMMA RAYS WITH MATTER

All calculations on shielding or analysis of shielding experiments, require a knowledge of the sources of the radiation. The (γ, γ) scattering processes, which alter the energy and the direction of the photon, are of primary interest in the discussion. However, the list of the gamma interaction sources are quite lengthy.

Gamma-ray sources:[1]

Prompt fission: some gamma radiation is emitted essentially in coincidence ($\tau < 1$ μ sec) with the fission process and this radiation is said to be prompt fission gamma radiation.

Short period fission: The radioactive fission products emit gamma rays which fall off rapidly with time after the fission. Most of this radiation occurs within the first 10 minutes, and is indistinguishable from the prompt gamma rays. This radiation is often referred as short period gamma rays.

Long period fission: The gamma rays emitted by fission products for a time greater than ten (10) minutes do not contribute appreciably to the core radiations during operation, but are important radiation sources after shut down. They are denoted as long period fission gamma rays.

Capture γ: Gamma rays emitted in the (n, γ) reaction are customarily referred to as capture gamma rays.

Gamma radiation from inelastic collision: During the scattering of neutrons by inelastic process, the compound nucleus formed loses its excitation energy by the emission of one or more gamma rays.

Reaction: The products of charged particle reactions induced by neutrons emits gamma rays immediately. These could be denoted as "Reaction Gamma's".

Activation: Many nuclear neutron reactions result in radioactive nuclei, which upon decay, emit gamma radiation. These are activation gamma rays.

Bremsstrahlung and annihilation: Electrons emitted from activated end products of nuclear reaction can produce gamma rays by electromagnetic process known as bremsstrahlung.

Positrons emitted from radioactive nuclei are annihilated by collisions with atomic electrons, resulting in the creation of two 0.511 - Mev photons.

However such events should be considered as a part of the gamma-ray interaction process rather than as an immediate source of gamma radiation.

Calculations of the attenuation of gamma rays would be relatively simple if the interaction processes were purely absorptive;

that is, if each collision resulted in the disappearance of a photon and we could mathematically correlate the process of absorption by Lambert's law[1] of absorption as represented in Equation (1). As a matter of fact, gamma ray interaction processes are not always absorptive. Photons often survive collision and change their direction and energy. The interaction is a pure scattering one. If a photon is absorbed in a pair production process, the resulting electron and positron give rise to secondary gammas at the annihilation processes - so Lambert's law does not fully explain the attenuated gamma rays.

Gamma ray interaction processes can be broadly divided into two distinct categories:

a) Primary and

b) Secondary.

Table I lists the processes:

TABLE I

Primary	Secondary
1. Compton Effect	1. Annihilation radiation
2. Photoelectric effect	2. Bremsstrahlung
3. Pair production	3. Fluorescence radiation
4. Coherent electron scattering (Raleigh)	
5. Bragg scattering	
6. Delbruck Scattering	

Primary (Continued)

7. Nuclear interaction
   a) Photo effect
   b) Scattering
8. Thompson scattering from the nucleus
9. Radiative corrections to lower processes

It is of primary importance to consider processes 1, 2 and 3 of Table I in shielding calculations for a source strength ranging between 20 Kev and 10 Mev.

Compton effect is a scattering process where a free electron in an atom alters the direction and energy of the incident photon.

The relationship between the incident photon wave length, scattered wave length, and the angle of scattering is described by the following relation:

$$\lambda' - \lambda = 1 - \cos\theta \qquad (2)$$

where λ' is the incident photon wave length expressed in units of compton wave length[1]

λ is the scattered wave length

θ is the angle of scattering.

Expressed in terms of energy,[1]

$$E' = \frac{E}{1 + E(1 - \text{Cos } \theta)}$$

where E' is the energy of the incident gamma

E is the energy of the gamma ray after collision

θ is the scattering angle.

Compton effect is pronounced at low energy and low atomic number (Z).

Pair Production:

In pair production all the energy of the incident photon is transformed to create two particles; an electron and a positron. The kinetic energy of the pair is equal to the photon energy less $2mc^2$, the rest mass of the pair. Pair production has a threshold at $2mc^2 = 1.022$ Mev, below which it cannot take place. A pair production cross section (α_{pp}) increases with the square of the atomic number (Z^2). It is also an increasing function of energy above threshold. Pair production interaction takes place when the atomic electrons or the charge on the nucleus presents a field of interaction[2]. It is a dominant interaction for high energy photons.

Photo electric effect:

In this process, a photon interacts with the atom as a whole. The photon is completely absorbed by an atom, which is ionised in the process; the ejected electron carries the vast majority of the photon energy. Hence photo electric effect is possible, if the photon energy is greater than the electron binding energy. The

ionised electron leaves the atom with kinetic energy equal to photon kinetic energy minus the electron binding energy. Since the energy of this electron is usually of the order of hundreds of kilo electron volts or less, it is absorbed in a short distance. The singly ionised atom becomes neutral by picking up a loose electron, or by readjusting its electrons. The x-radiation (called fluorescent radiation) which usually accompanies atomic electron readjustments is often replaced by emission of an Auger electron [2] (an electron ejected from a higher shell by the x-ray).

For low-energy photons or energy closest to the K-shell and high Z media, the photo electric effect is most favored. In Table I, interaction process itemized 6 through 9 can be neglected with the remark that below 10 Mev contribution of all of these are negligible compared to first three process (1 through 3)[3].

Multiple Bragg scattering is prominent in microcrystallites and affects the measurement in absorption coefficients in good geometry. It is a type of coherent scattering among atoms or molecules where incident photon wave-length is much greater than the spacing of electrons. It can be neglected between the energy range 0.5 and 10 Mev in such media as water, iron, concrete, aluminum, lead, uranium or zinc[3].

Coherent or Raleigh scattering:

This type of scattering is of greatest significance when the wavelength of the incident electromagnetic radiation is long

compared to the electron spacing. For low energy photons the target electron must, generally, be considered as bound. This fact makes possible a type of scattering from the electrons of an atom, not as individual electrons, but as a member of a group. This can be further visualized as the scattering of the incident photon beam by the spherical electron distribution about the atom. All things considered, this type of scattering makes its relative contribution to the total attenuation coefficients for Z-materials in the span 25 to 75. At high energies its importance decrease tremendously[3].

Annihilation:

In this process, two photons of about 50 Kev result from the annihilation of positrons created in pair production. It has been indicated that the annihilation radiation has a much more profound effect upon the scattered photon spectrum below 0.5 Mev. The effect of annihilation radiation production on gamma-ray attenuation is most important for high energy sources (10 Mev) and medium weight elements ($Z \sim 50$)[3]. Annihilation radiation can contribute between 5 to 6% to the computed dose which, for all practical purposes, can be neglected.

Fluorescence radiation:

The fluorescence radiation is the result of the emission of an atomic electron through the photo effect. Such x-ray emission can be reasonably energetic and abundant only in heavy elements. For a

very low-energy photon source in a heavy medium, such as lead, the contribution from such interaction in photon scattering could be around 2 to 3%[3].

Bremsstrahlung:

Bremsstrahlung may be described as the radiation produced by charged particles as they decelerate by collision with atoms. The charged particles are the electrons resulting from pair production and photoelectric effect or the recoil electrons from the Compton scattering. Unlike annihilation or fluorescence radiation, Bremsstrahlung results in shower production-by creating or ejecting new electrons which in turn give rise to secondary photons by Bremsstrahlung, and so on until the initial energy is completely lost. Most of the photons produced in Bremsstrahlung are very much lower in energy than the electron. Shower formation is important only above 10 Mev. Bremsstrahlung increases rapidly with atomic number above 10 Mev[3].

After elimination of all other primary and secondary interactions, only three major interaction processes remain. Two are treated as purely absorptive - photoelectric effect and pair production. The third, Compton scattering from free electrons, is the only scattering process.

Further repetition would be superfluous since excellent references exist which discuss these interactions in greater detail[1], [2], [4], [5], [6].

CHAPTER 3

MATHEMATICAL DESCRIPTION OF THE PROBLEM

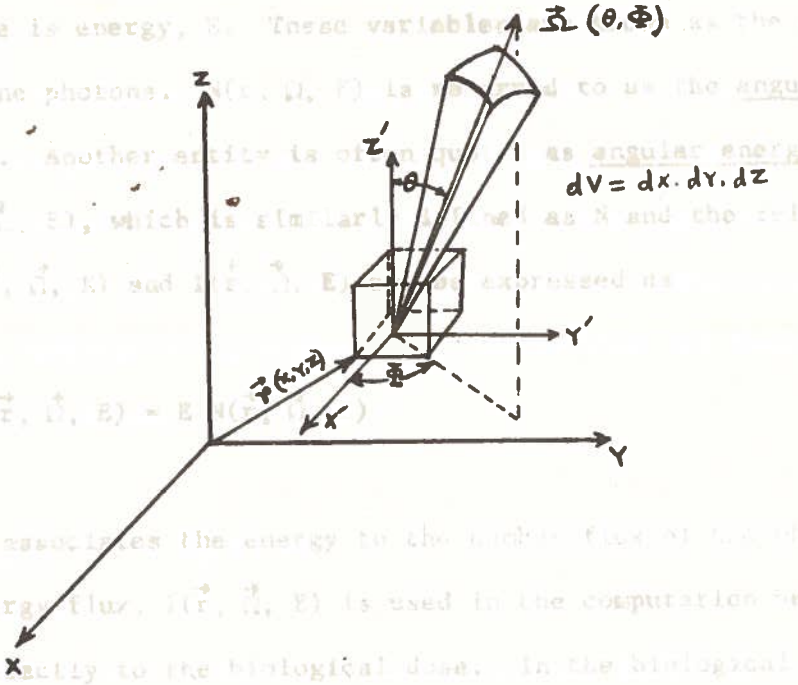

Figure 1

Definition of associated terms in transport equation

Gamma Ray transport theory deals with gamma photons as a function of direction as well as energy. The number flux of photon $N(\vec{r}, E, \vec{\Omega}, t)$ is defined as number of photons/(cm^2)(sec)(Mev)(ster) as position $\vec{r}$ with energy E, at direction $\vec{\Omega}$ and time t. For the steady state the time dependence is eliminated. The function $N(\vec{r}, \Omega, E)d\Omega dE$ denotes as the number of photons between energy E and E + dE, moving in the direction of unit vector $\vec{\Omega}$ in the element of solid angle $d\Omega$, which cross in unit time through a

a unit area located at the position $\vec{r}$ whose normal is in the direction $\vec{\Omega}$. N is generally dependent on six variables - three of position vector $\vec{r}$ (x, y, z), two of directional vector $\vec{\Omega}$ (θ, φ) and the last one is energy, E. These variables are known as the phase space for the photons. $N(r, \Omega, E)$ is referred to as the angular number flux. Another entity is often quoted as angular energy flux, $I(\vec{r}, \vec{\Omega}, E)$, which is similarly defined as N and the relationship between $N(\vec{r}, \vec{\Omega}, E)$ and $I(\vec{r}, \vec{\Omega}, E)$ can be expressed as

$$I(\vec{r}, \vec{\Omega}, E) = E\, N(\vec{r}, \vec{\Omega}, E) \qquad (2)$$

$I(\vec{r}, \vec{\Omega}, E)$ associates the energy to the number flux of the photons. Angular energy flux, $I(\vec{r}, \vec{\Omega}, E)$ is used in the computation because it leads directly to the biological dose. In the biological shielding measurement, emphasis lies in the energy of gamma rays received by a detector, ion chamber, which can discriminate in energy but not the angle of incidence. The measureable quantity is

$$I_o(\vec{r}, E) = \int_{\vec{\Omega}} E\, N(\vec{r}, \vec{\Omega}, E)\, d\vec{\Omega} \qquad (3)$$

$$= \int_{\vec{\Omega}} I(\vec{r}, \vec{\Omega}, E)\, d\vec{\Omega} \qquad (4)$$

whose unit can be expressed as

Mev. no. of photons/(Mev)(cm^2)sec

It is worthwhile to mention that

$$N_o(\vec{r}, E) = \int_{\vec{\Omega}} N(\vec{r}, \vec{\Omega}, E)\, d\Omega \qquad (5)$$

which can be interpreted as the number of photons, of energy E in unit energy range, incident per unit time, upon a differential sphere of unit cross sectional area located at the position $\vec{r}$.

$N_o(\vec{r}, E)$ is often quoted as the differential number flux and $I_o(\vec{r}, E)$ is referred to as differential energy flux and these two functions are related as follows:

$$I_o(\vec{r}, E) = E\, N_o(\vec{r}, E)$$

It is necessary to review few preliminaries before going into Boltzmann's integro-differential equation for gamma transport. For certain source geometries, the variables as described before can be reduced due to the geometrical symmetry. Such would be the case in point isotropic or infinite plane uniform source, where the photon distributions is a function of one position variable, either the radial distance from the point isotropic source or the perpendicular distance from the plane infinite uniform source. Only one direction variable is involved, ω , the cosine of the angle between the photon direction and the radius vector from the source, as shown in Figure 2.

The element of solid angle $d\vec{\Omega}$ is equal to the surface area intercepted on a sphere by cones of half angles θ and $\theta + d\theta$, divided by the square of the radius of the sphere.

The area intercepted by the cone:

$$dA = 2\pi\ (r \sin\theta)(r d\theta)$$

$$d\vec{\Omega} = \frac{dA}{r^2} = \frac{2\pi\ r^2 \sin\theta\ d\theta}{r^2}$$

$$\int d\vec{\Omega} = \vec{\Omega} = 2\pi \int_0^{\pi} \sin\theta\ d\theta = 4\pi$$

Photon balance

For a steady state,

[Rate of loss of photons from the differential Control volume, dV] = [Rate of Gain of photons in the differential Control volume, dV.]

Losses of photons:

1. Those scattering out
2. Those absorbed in the medium
3. Convective flow of photon associated with the physical passage of photons out of dV.

The attenuation coefficient $\mu(E)(cm^{-1})$ accounts for the probability of total loss in scattering out and absorption, then

$$\mu(E)N(\vec{r}, \vec{\Omega}, E)\ dE\ d\Omega\ dv \quad \text{\# photons/sec}$$

is the loss due to the above mentioned effects 1 and 2 in Table 1.

Consider Figure 2(b). We are interested in the flow of photon at $\vec{\Omega}$ passing through an area of 1 cm^2 normal to $\vec{\Omega}$.

The no. of photons/sec passing through the area ABCD is

$$N(\vec{r}, \vec{\Omega}, E)\ d\Omega\ dE(\Delta x\ \Delta y)\ \vec{\Omega}\cdot k$$

where $\vec{\Omega}\cdot k = \vec{\Omega}_Z = \cos\theta$

The no. of photons/sec passing through the area EFGH is

$$\left[N(\vec{r}, \vec{\Omega}, E) + \frac{\Delta N}{\Delta Z}(\vec{r}, \vec{\Omega}, E)\ \Delta Z\right]\ d\Omega\ dE(\Delta x\cdot\Delta y)\ \vec{\Omega}\cdot k$$

The net flow in z-direction is the difference of the above terms.

$$\lim_{\Delta z \to 0} \frac{\Delta N}{\Delta z}(\vec{r}, \vec{\Omega}, E)\ d\Omega \cdot dE\ (\Delta x \Delta y \Delta z)\ \vec{\Omega}\cdot k$$

As the differential increments approaches zero, the flow term in z direction becomes:

$$\vec{\Omega}.k\ \frac{\partial N}{\partial Z}(\vec{r}, \vec{\Omega}, E)\ dv$$

where the differential volume is: $dv = \Delta x \Delta y \Delta z$.

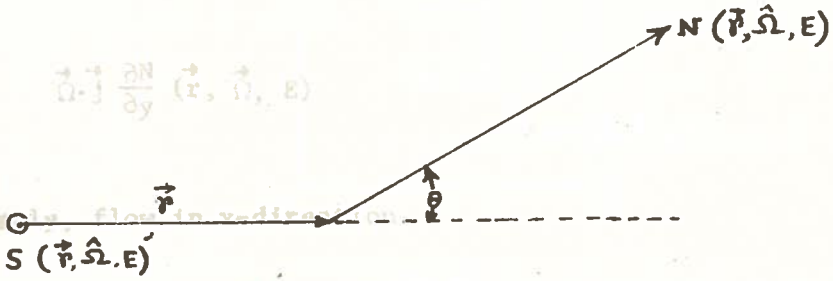

Figure 2

$\omega = \cos\theta$

Consider photons in a differential volume (dv(dx·dy·dz) at a position $\vec{r}$, between the energy range E and E + dE moving in the direction $\vec{\Omega}$. Consider those photons between the solid angle, $\vec{\Omega}$ and $\vec{\Omega} + d\vec{\Omega}$. The differential solid angle can be pictured as in Figure 2

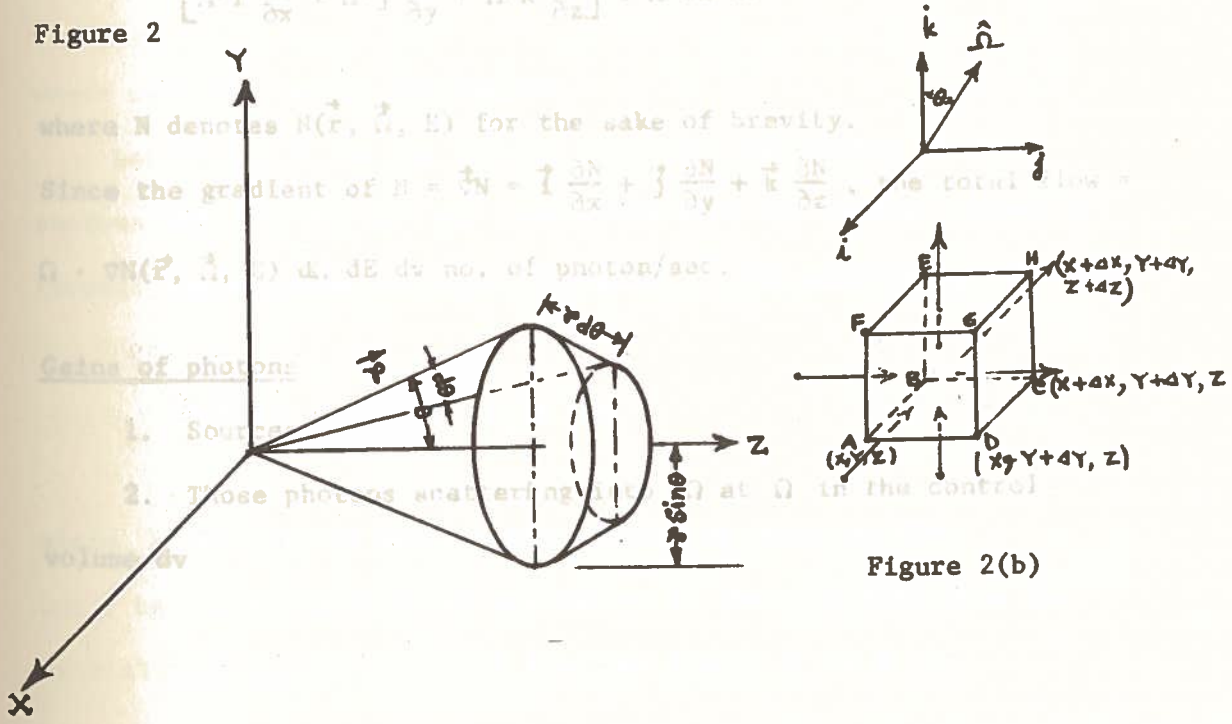

Figure 2(a)

Figure 2(b)

Similarly, flow in y direction

$$\vec{\Omega}\cdot\vec{j}\,\frac{\partial N}{\partial y}\,(\vec{r},\,\vec{\Omega},\,E)$$

Similarly, flow in x-direction

$$\vec{\Omega}\cdot\vec{i}\,\frac{\partial N}{\partial x}\,(\vec{r},\,\vec{\Omega},\,E)$$

The combined three dimentional convective flow (no. of photon/sec) becomes

$$\left[\vec{\Omega}\cdot\vec{i}\,\frac{\partial N}{\partial x}+\vec{\Omega}\cdot\vec{j}\,\frac{\partial N}{\partial y}+\vec{\Omega}\cdot\vec{k}\,\frac{\partial N}{\partial z}\right]\,d\,\Omega\,dE\,dv$$

where N denotes $N(\vec{r},\,\vec{\Omega},\,E)$ for the sake of brevity.

Since the gradient of $N = \vec{\nabla}N = \vec{i}\,\frac{\partial N}{\partial x}+\vec{j}\,\frac{\partial N}{\partial y}+\vec{k}\,\frac{\partial N}{\partial z}$, the total flow = $\Omega\cdot\nabla N(\vec{r},\,\vec{\Omega},\,E)\,d\Omega\,dE\,dv$ no. of photon/sec.

<u>Gains of photons</u>

1. Sources.
2. Those photons scattering into $d\Omega$ at Ω in the control volume dv

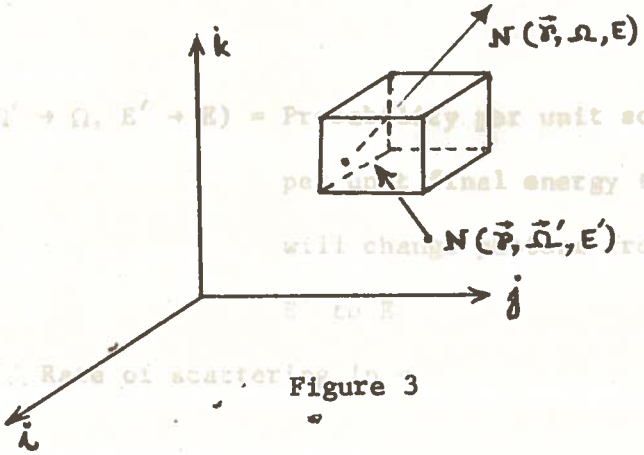

Figure 3

Since the source term has the unit of no. of photons/Mev. ster. cm^3. sec., the total number of photons emitted per sec from the source inside the control volume dv within dE range at E and $d\Omega$ at $\vec{\Omega}$ is

$$S(\vec{r}, \vec{\Omega}, E)\, d\Omega\, dE\, dv,$$

whose unit is: no of photons/sec.

Let $\vec{\Omega}'$ and E' be the initial direction and energy of the photons $N(\vec{r}, \vec{\Omega}', E')\, dE'\, d\Omega'$ scattering into the control volume dv through the differential solid angle $d\Omega'$. The scattering rate is

$$N(\vec{r}, \vec{\Omega}, E)\, dE'\, d\Omega'\, \Sigma_s(E')dv$$

where $\Sigma_s(E')$ is the macroscopic scattering eross section.
Let P be the probability of scattering of those photons from the initial direction Ω' to Ω and from the energy E' to E

$P(\Omega' \rightarrow \Omega, E' \rightarrow E)$ = Probability per unit solid angle $(\vec{\Omega})$ per unit final energy that a scatterer will change photons from Ω' to Ω, and E' to E

$\therefore$ Rate of scattering in =

$$\int_{E'} \int_{\Omega'} N(\vec{r}\cdot\vec{\Omega}', E')\, dE'\, d\Omega'\, \Sigma_s(E')dv\, P(\Omega' \rightarrow \Omega, E' \rightarrow E)\, d\Omega\, dE$$

Equating all gain and loss terms, we can write the photon balance equation as:

$$S(\vec{r}, \vec{\Omega}, E)\, d\Omega dE\, dv + \int_{E'} \int_{\Omega'} N(\vec{r}, \Omega', E')\, dE'\, d\Omega'\, \Sigma_s(E')\, dv$$

$$P(\Omega' \rightarrow \Omega, E' \rightarrow E)\, d\,\Omega\, dE$$

$$= \Omega\cdot\nabla N\,(\vec{r}, \vec{\Omega}, E)\, d\Omega\, dE\, dv + \mu(E)\, N(\vec{r}, \vec{\Omega}, E)\, d\Omega\, dE\, dv$$

or,

$$S(\vec{r}, \vec{\Omega}, E) + \int_{E'} \int_{\Omega'} N(\vec{r}, \vec{\Omega}', E')\, \Sigma_s(E')P(\vec{\Omega}' \rightarrow \vec{\Omega}, E' \rightarrow E)\, dE'\, d\Omega' \qquad (6)$$

$$- \vec{\Omega}\cdot\nabla N\,(\vec{r}, \vec{\Omega}, E) - \mu(E)\, N\,(\vec{r}, \vec{\Omega}, E) = 0$$

Equation (6) is the celebrated time-independent Boltzmann's Gamma

transport equation.

For the sake of brevity, let's rewrite the Equation (6) as

$$S + \int_{E'} \int_{\Omega'} N' \Sigma_s' P \, dE' \, d\Omega' - \Omega \cdot \vec{\nabla} N - \mu N = 0 \qquad (7)$$

In shielding study we are interested in the energy flux as shown in (2), $I = EN$. Multiplying (7) by E,

$$SE + E \int_{E'} \int_{\Omega'} \frac{E'}{E'} N' \Sigma_s' P \, dE' \, d\Omega' - E\Omega \cdot \vec{\nabla} N - \mu EN = 0$$

Rearranging,

$$\Omega \cdot \vec{\nabla} I + \mu I = SE + \int_{E'} \int_{\Omega'} I' \frac{E}{E'} \Sigma_s' P \, dE' \, d\Omega' \qquad (8)$$

Now, consider the quantity associated with the scattering probability

$$\int_{E} \int_{\Omega} \Sigma_s' P(\vec{\Omega}' \rightarrow \vec{\Omega}, E' \rightarrow E) \, d\Omega \, dE \qquad (9)$$

$\therefore$ where $\Sigma_s(E') = \Sigma_s' \qquad (10)$

By definition of the probability function $\int_{E} \int_{\Omega} P(\vec{\Omega}' \rightarrow \vec{\Omega}, E' \rightarrow E) \, d\Omega dE = 1$

Differentiating (10) with respect to Ω

$$\frac{d\,\Sigma_s(E')}{d\Omega} = \int_E \Sigma_s(E')\; P\;(\Omega' \rightarrow \rightarrow,\; E' \rightarrow E)\; dE \qquad (11)$$

where $\frac{d\,\Sigma_s(E')}{d\Omega}$ is known as the differential cross-section.

Rewriting (10), $\Sigma_s(E') = \Sigma_s{}' = \eta\,\sigma_s(E')$ where η is the electron density in the medium, electron/cm^3, and sometimes referred to as the number of scatterer. The differential cross-section is then,

$$\frac{d\,\Sigma_s(E')}{d\Omega} = \eta\,\frac{d\,\sigma_s(E')}{d\Omega}$$

where $\frac{d\,\sigma_s(E')}{d\Omega}$ is known as the Klein-Nishina scattering cross-section. The relationship between the initial energy E', final energy E and the scattering angle Ψ in compton scattering is related by

$$E = \frac{E'}{1 + \frac{E'}{m_o c^2}\,(1 - \cos\Psi)} \qquad (12)$$

where m_o is the rest mass of electron and c is the relativistic speed of electron.

Expressing E and E' in terms of $m_o c^2$ unit, we have,

$$\frac{E}{m_o c^2} = \frac{E'/m_o c^2}{1 + \frac{E'}{m_o c^2}\,(1 - \cos\Psi)} \qquad (13)$$

$$\text{also, } E = h\nu = h\frac{c}{\lambda} = \frac{h\cdot(m_o c)c}{\lambda\, m_o c} = \frac{m_o c^2}{\frac{\lambda}{(h/m_o c)}}$$

$$\therefore\ E = \frac{m_o c^2}{\frac{\lambda}{(h/m_o c)}} \qquad (14)$$

where

$$m_o c^2 = 0.511 \text{ Mev}$$

$$\frac{h}{m_o c} = 0.02426^{o}\text{A}$$

$$\frac{\lambda}{h/m_o c} = \text{compton wave length}$$

Now if we express λ in the dimensionless compton's wave length unit:

$$E = \frac{0.511}{\lambda}$$

Substituting (14) in (13)

$$\frac{1}{\left(\frac{\lambda}{h/m_o c}\right)} = \frac{\frac{1}{\left(\frac{\lambda'}{h/m_o c}\right)}}{1 + \frac{1}{\frac{\lambda'}{(h/m_o c)}}(1 - \cos\Psi)}$$

and changing λ into Compton's unit:

$$\frac{1}{\lambda} = \frac{\frac{1}{\lambda'}}{1 + \frac{1}{\lambda'}(1 - \cos \Psi)}$$

which gives

$$\lambda - \lambda' = 1 - \cos \Psi \qquad (15)$$

where λ is the scattered wave length in Compton's unit and λ' is the incident photon wave length in Compton's unit.

Simplification of scattering - in term

Rewriting scattering-in term in (8), $\frac{d\Sigma_s}{d\Omega}(E')$ in (11) and (15)

$$\int_{E'} \int_{\Omega'} \Sigma_s' \, P(\Omega' \to \Omega, E' \to E) \frac{I'}{E'} E \, d\Omega' \, dE' \qquad (16)$$

$$\Sigma_s' \int P(\Omega' \to \Omega, E' \to E) \, dE = \eta \frac{d\sigma_s(E')}{d\Omega} \qquad (17)$$

$$\lambda - \lambda' = 1 - \cos \Psi \qquad (18)$$

For $0 \le \Psi \le \pi$, $\lambda' \le \lambda \le \lambda' + 2$ (19)

Since the unique relationship exists between the scattering angle and the wave length λ, we can express (18) in terms of a δ function

$$\therefore \int_{E'}^{E} \delta\,(1 + \lambda' - \lambda - \cos\Psi)\, d\lambda = -1 \qquad (20)$$

The minus sign is due to the fact that $E' > E$.

Multiplying (17) and (20)

$$-\Sigma_s' \int P(\vec{\Omega}' \rightarrow \vec{\Omega},\, E' \rightarrow E)\, dE = \eta \frac{d\sigma_s(E')}{d\Omega} \int_{\text{Low E}}^{\text{High E}} \delta(1 + \lambda' - \lambda - \cos\Psi) d\lambda \qquad (21)$$

Using the dimensionless relationship $E = \frac{1}{\lambda}$

$$dE = -\frac{d\lambda}{\lambda^2} \qquad d\lambda = -\frac{dE}{E^2} \qquad (22)$$

Differentiating (21)

$$-\Sigma_s' P = \eta \frac{d\sigma_s(E')}{d\Omega} \delta(1 + \lambda' - \lambda - \cos\Psi) \frac{1}{E^2} \qquad (23)$$

Substituting (23) into (16)

$$\int_{E}^{E'} \int_{\Omega'} \eta \frac{d\sigma_s(E')}{d\Omega} \delta(1 + \lambda' - \lambda - \cos\Psi) \frac{1}{E^2} \cdot \frac{I'}{E'E}\, d\Omega'\, dE' \qquad (24)$$

But $dE' = -\frac{d\lambda'}{\lambda'^2}$ and

$$\frac{dE'}{EE'} = -\frac{\lambda\lambda'}{\lambda'^2} d\lambda'$$

Rearranging (24) and reverse the integration limits:

$$\int_{E'}^{E} \int_{\Omega'} \eta \frac{d\sigma_s(E')}{d\Omega} \delta(1 + \lambda' - \lambda - \cos\Psi) I' \frac{\lambda}{\lambda'^2} d\lambda' \qquad (25)$$

Defining $k(\lambda', \lambda) = 2\pi \frac{\lambda}{\lambda'^2} \frac{d\sigma_s(\lambda')}{d\Omega}$ for $\lambda' \leq \lambda \leq \lambda' + 2$ (26)

Equation (25) can be rewritten as

$$\int_{\text{Low } \lambda}^{\text{High } \lambda} \int_{\Omega'} \eta I' \frac{k(\lambda', \lambda)}{2\pi} \delta(1 + \lambda' - \lambda - \cos\Psi) d\lambda' \, d\Omega' \qquad (27)$$

For a single Compton scattering, the low limit of λ is λ' and the high limit of λ is $\lambda' + 2$.

$$\int_{\lambda'}^{\lambda' + 2} \int \eta I' \frac{k(\lambda', \lambda)}{2\pi} \delta(1 + \lambda' - \lambda - \cos\Psi) \, d\lambda' \, d\Omega' \qquad (28)$$

The Klein-Nishina scattering cross-section is given by

$$\frac{d\sigma_s}{d\Omega} = \frac{r_o^2}{2} \left(\frac{E}{E'}\right)^2 \left[\frac{E}{E'} + \frac{E'}{E} - \sin^2\Psi\right] \qquad (29)$$

with a unit of cm^2 since $\cos\Psi = 1 + (\lambda' - \lambda)$, it leads to

$$\sin^2\Psi = -\left[2(\lambda' - \lambda) + (\lambda' - \lambda)^2\right]$$

Rearranging (26) and (29)

$$k(\lambda', \lambda) = \pi r_o^2 \left(\frac{\lambda'}{\lambda}\right)^2 \left[\frac{\lambda'}{\lambda} + \frac{\lambda}{\lambda'} + 2(\lambda' - \lambda) + (\lambda' - \lambda)^2\right] \frac{cm^2}{(electron)(ster)} \tag{30}$$

The transport equation (8), with the changes made in the scattering-in term according to equation (28), can be rewritten as,

$$\Omega \cdot \nabla I + \mu(\lambda) I = SE + \int_0^{\lambda} \int_{\Omega'} \eta I' \frac{k(\lambda' \lambda)}{2\pi} \delta(1 + \lambda' - \lambda - \cos \Psi) d\lambda' \, d\Omega' \tag{31}$$

Simplification of the photon flow term

Consider the term $\Omega \cdot \nabla I$ in (31)

$$\Omega \cdot \nabla I = \nabla \cdot \Omega \, I \, (\vec{r}, \vec{\Omega}, \lambda)$$

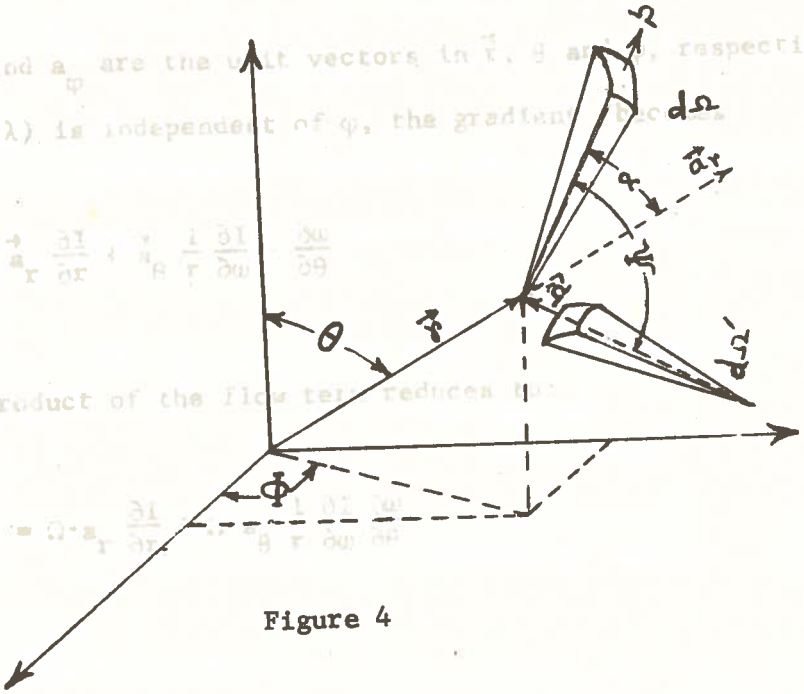

Figure 4

Let α be the angle between the photon direction Ω and the radius vector $\vec{r}$. For isotropic source $I(\vec{r}, \vec{\Omega}, \lambda)$ is a function of $\vec{r}$ and α only in a spherical geometry. α depends on $\vec{r}$ and hence θ. So $I(r, \Omega, \lambda)$ is independent of the angle φ.

$$\Omega \cdot a_r = \cos \alpha$$

Let $\cos \alpha = \omega$ and $dI/d\varphi = 0$

we can change the direction variable, Ω, in $I(\vec{r}, \vec{\Omega}, \lambda)$ to ω, $I(\vec{r}, \omega, \lambda)$.

In spherical co-ordinate system the gradient is given as

$$\nabla I(r, \Omega, \lambda) = \vec{a}_r \frac{\partial I}{\partial r} + \vec{a}_\theta \frac{1}{r} \frac{\partial I}{\partial \theta} + \vec{a}_\varphi \frac{\partial I}{r \sin \theta \partial \varphi}$$

where a_r, a_θ and a_φ are the unit vectors in $\vec{r}$, θ and φ, respectively. Since $I(r, \omega, \lambda)$ is independent of φ, the gradient becomes

$$\vec{\nabla} I = \vec{a}_r \frac{\partial I}{\partial r} + \vec{a}_\theta \frac{1}{r} \frac{\partial I}{\partial \omega} \cdot \frac{\partial \omega}{\partial \theta}$$

And the dot product of the flow term reduces to:

$$\Omega \cdot \nabla I = \Omega \cdot a_r \frac{\partial I}{\partial r} + \Omega \cdot a_\theta \frac{1}{r} \frac{\partial I}{\partial \omega} \frac{\partial \omega}{\partial \theta}$$

where $\Omega \cdot \vec{a}_r = \cos \alpha = \omega$. Differentiating ω with respect to θ we have

$$* \quad \frac{\partial \omega}{\partial \theta} = \frac{\partial}{\partial \theta} (\Omega \cdot a_r) = \Omega \cdot \frac{\partial a_r}{\partial \theta} = \Omega \cdot a_\theta$$

But $\Omega \cdot a_\theta = \cos (90 + \alpha) = - \sin \alpha = - (1 - \omega^2)$, the second term of the gradient reduces to $\frac{1 - \omega^2}{r} \frac{\partial I}{\partial \omega}$.

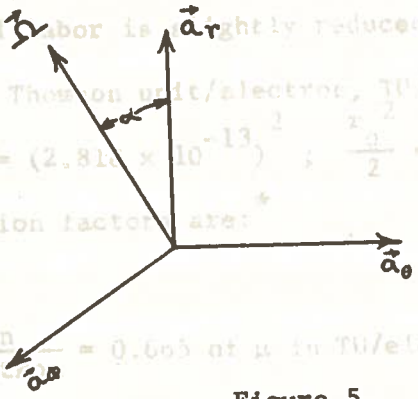

Figure 5

Thus, for a point monoenergetic source at the origin, of strength 1 photon/sec of energy E', the transport equation can be written as

$$\omega \frac{\partial I(r, \omega, \lambda)}{\partial r} + \frac{1 - \omega^2}{r} \frac{\partial I}{\partial \omega} + \mu(\lambda) I = S + \int_0^{\lambda} \int_{\text{over } 4\pi \text{ solid angle}} \eta \, I'(r, \Omega', \lambda')$$

$$k(\lambda', \lambda) \frac{\delta(1 + \lambda' - \lambda - \cos \Psi)}{2\pi} d\lambda' \, d\Omega' \qquad (32)$$

* Ref: Page 24, NYO3075

The isotropic source term at $\vec{r}$ can be expressed as:

$$S(\vec{r}, \lambda, \Omega) = [\, S(\lambda)4\pi \,]\, [\, \delta(r)/4\pi r^2 \,]$$

The total energy flux, I, consists of unscattered, I^o, and scattered angular energy flux, I^s. For the sake of brevity, let's denote $\eta \frac{k(\lambda', \lambda)}{2\pi}$ as the Klein-Nishina function which has the unit of $\frac{\text{electron}}{\text{cm}^3} \frac{\text{cm}^2}{(\text{ster})\ \text{electron}}$. Because of the multiplicative factor, $\frac{8\pi}{3} \left(\frac{e^2}{m_o r_o^2}\right)^2$ in the Klein-Nishina formula, it has been suggested the the numerical labor is slightly reduced if absorption coefficients are expressed in Thomson unit/electron, TU/electron. In Equation (30) $r_o^2 = (e^2/m_o c^2) = (2.818 \times 10^{-13})^2$; $\frac{r_o^2}{2} = 0.0397$ barn.

The conversion factors are:*

$$\mu, \frac{\text{barn}}{\text{electron}} = 0.665 \text{ of } \mu \text{ in TU/electron}$$

$$\mu, \frac{\text{cm}^2}{\text{gm}} = 0.4005 \frac{Z}{A} \text{ of } \mu \text{ in TU/electron}$$

$$\mu, \text{cm}^{-1} = 0.4005\ \rho \frac{Z}{A} \text{ of } \mu \text{ in TU/electron } \rho \text{ is the density, } \frac{\text{gms}}{\text{cm}^3}$$

In Thomson unit/electron,

$$k(\lambda', \lambda) = \frac{3}{8} \left(\frac{\lambda'}{\lambda}\right) \left[\frac{\lambda}{\lambda'} + \frac{\lambda'}{\lambda} + 2(\lambda' - \lambda) + (\lambda' - \lambda)^2\right] \qquad (33)$$

* NYO 3075, pp. 10

for $\lambda' \leq \lambda \leq \lambda' + 2$

$= 0$, otherwise.[14]

Finally, for a plane uniform source, the symmetric property further reduces the transport equation to

$$\omega \frac{\partial I}{\partial Z}(Z, \omega, \lambda) + \mu(\lambda) I(Z, \omega, \lambda) = \int_0^{\lambda} \int_{4\pi} I(Z, \omega', \lambda') k(\lambda', \lambda) \frac{\delta(1 + \lambda' - \lambda - \cos \Psi)}{2\pi} d\lambda' \, d\Omega' + SE(\lambda, \omega) \, \delta(Z) \tag{34}$$

Before converting Equation (34) into inter-linked, doubly indexed equation, let's look at the source term for an infinite plane, isotropic, monoenergetic source, emitting 1 photon/(cm^2)(sec), located at z = 0

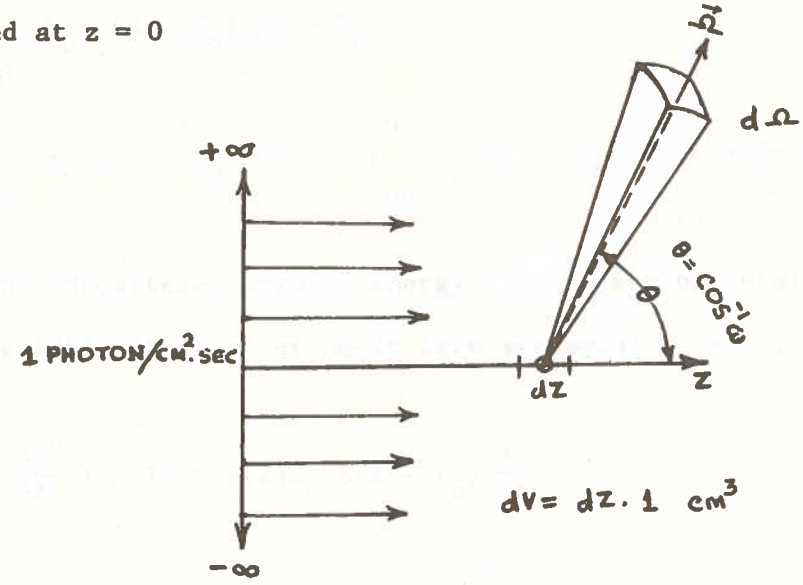

Figure 6
Infinite Uniform Plane Source

$$S(Z, \omega, \lambda) = S(\lambda, \omega)\ \delta(Z)\ \delta(E - E_o)\ \frac{1}{4\pi}\ \frac{\text{Reaction}}{(cm^3)(sec)(Mev)(ster)}$$

$$= 1\ \delta(Z)\ \delta(E - E_o)\ \frac{1}{4\pi}\ \frac{\text{Reaction}}{(cm^3)(sec)(Mev)(ster)}$$

$$E\ S(Z, \omega, \lambda) = \frac{S(\lambda, \omega)\ E_o\ \delta(E - E_o)}{4\pi}\ \delta(Z)$$

$$= 1\ \frac{E_o\ \delta(Z)\ \delta(E - E_o)}{4\pi}$$

$E_o\ \delta(E - E_o)$ in the source term can be replaced by its equivalent term,

$$\lambda_o\ \delta(\lambda - \lambda_o) \tag{35}$$

Equation (35) is Dirac-Delta Function.

$$\int_0^\infty E_o\ \delta(E - E_o)\ dE = E_0$$

$$\int_0^\infty \lambda_o\ \delta(\lambda - \lambda_o)\ dE = -\int_{00}^{0} \lambda_o\ \delta(\lambda - \lambda_o)\ \frac{d\lambda}{\lambda^2} = +\frac{1}{\lambda_o} = E_o$$

The unscattered angular energy flux I^o can be obtained by solving (34) with scattering-in term set equal to zero.

$$\omega\ \frac{\partial I^o}{\partial Z} + \mu\ I^o = \delta(Z) E_o\ \delta(E - E_o)\ \frac{1}{4\pi} \tag{36}$$

Multiply (36) by the integrating factor $e^{\frac{\mu Z}{\omega}}$, and integrate, we have

$$I^{o} e^{\frac{\mu Z}{\omega}} = \int_{0}^{\infty} e^{\frac{\mu Z}{\omega}} \delta(Z) E_{o} \delta(E - E_{o}) \frac{1}{4\pi} dZ$$

Using the boundary condition that when $|Z|$ approaches ∞, $I^{o}(Z, \omega, \lambda) = 0$

$$I^{o}(Z, \omega, E) = \frac{E_{o} \delta(E - E_{o})}{4\pi |\omega|} e^{-\frac{\mu Z}{\omega}} \int_{0}^{\infty} e^{\frac{\mu Z}{\omega}} \delta(Z) dZ \qquad (37)$$

$$= \frac{E_{o} \delta(E - E_{o})}{4\pi |\omega|} e^{-\frac{\mu Z}{\omega}}$$

Since we are concerned with the limit of Z between 0 to ∞ a new function called Heaviside function which is defined as

$$H(Z) = 1 \ , > 0$$

$$= 0 \ , < 0$$

Equation (37) is simply,

$$I^{o}(Z, \omega, \lambda) = \frac{\lambda_{o} \delta(\lambda - \lambda_{o})}{4\pi |\omega|} e^{-\frac{\mu Z}{\omega}} H(\omega Z) \qquad (38)$$

Integrating over all solid angles:

$$I^{o}(Z, E) = \int I^{o}(Z, E, \Omega) \, d\Omega$$

However,

$$\int d\Omega = 2\pi \int_{-1}^{+1} d\omega \tag{39}$$

$$\therefore \quad I^{o}(Z, E) = 2\pi \int_{-1}^{+1} I^{o}(Z, E, \omega)\, d\omega \tag{40}$$

Substitute (38) in (40) and since we are interested in the positive z-axis only,

$$I^{o}(Z, E) = 2\pi \int_{-1}^{+1} \frac{E_o\, \delta(E - E_o)}{4\pi |\omega|} e^{-\frac{\mu Z}{\omega}} H(\omega Z)\, d\omega$$

$$= \frac{E_o}{2} \int_{-1}^{+1} \frac{e^{-\frac{\mu Z}{\omega}} H(\omega Z)}{|\omega|}\, d\omega \tag{41}$$

Now, let $y = \frac{\mu Z}{\omega}$ where $\mu = \mu(E_o)$

$$dy = -\mu Z \frac{d\omega}{\omega^2}$$

or, $$d\omega = -\frac{\omega^2}{\mu Z}\, dy$$

The integration limits are

$$\omega = 0 , \quad y = \infty$$

$$\omega = 1 , \quad y = \mu Z$$

Equation (41) can be written as,

$$I^{o}(Z, E) = -\frac{E_o}{2} \int_{\infty}^{\mu Z} \frac{e^{-\frac{\mu Z}{\omega}}}{|\omega|} \cdot \frac{\omega^2}{\mu Z}\, dy$$

$$= +\frac{E_o}{2} \int_{\mu Z}^{\infty} \frac{e^{-y}}{y}\, dy$$

whose lower integral limit applies for positive z only. To avoid delta function in the Boltzmann's Transpart equation for machine computation the unscattered energy angular flux can be subtracted from the total angular energy flux;

$$I = I^{o} + I^{s}$$

Subtracting (36) from (34), we have

$$\omega \frac{\partial I^{s}}{\partial Z}(Z', \lambda, \omega') + \mu(E) I^{s}(Z, \lambda, \omega) = \int_{4\pi} d\Omega' \int_{\lambda'=0}^{\lambda} d\lambda' \left[I^{o}(Z,\lambda',\omega')\right.$$

$$\left. + I^{s}(Z, \lambda', \omega')\right] \eta \frac{k(\lambda', \lambda)}{2\pi} \delta(1 + \lambda' - \lambda - \cos \Psi) \qquad (42)$$

where $I^{o}(Z, \lambda', \omega')$ is replaced by (38). The first term at the right hand side of (42) becomes:

$$\int_{4\pi} d\Omega' \int_{\lambda'}^{\lambda} d\lambda' \lambda_o \, \delta(\lambda' - \lambda_o) \frac{e^{-\frac{\mu Z}{\omega'}}}{4\pi|\omega'|} H(\omega' Z) \eta \frac{k(\lambda', \lambda)}{2\pi} \delta(1+\lambda'-\lambda-\cos\Psi)$$

$$= \int_{4\pi} d\Omega' \lambda_o \frac{e^{-\frac{\mu_o Z}{\omega'}}}{4\pi|\omega'|} H(\omega' Z) \, \eta \, \frac{k(\lambda_o, \lambda)}{2\pi} \delta(1 + \lambda_o - \lambda - \cos\Psi) \qquad (43)$$

Following relationship has been used in (42)

$$\int_{\text{all } x} dx \, f(x) \, \delta(X - a) \, \delta(x - b) = f(a) \, \delta(a - b)$$

Final form of Equation (42) aimed in solution of the problem is given by

$$\omega \frac{\partial I^s}{\partial Z}(Z, \lambda, \omega) + \mu(E) I^{s'}(Z, \lambda, \omega) = \int_{4\pi} d\Omega' \lambda_o \frac{e^{-\frac{\mu_o Z}{\omega'}}}{4\pi|\omega'|} H(\omega' Z) k(\lambda_o, \lambda)$$

$$\delta(1 + \lambda_o - \lambda - \cos\Psi) \qquad (44)$$

CHAPTER 4

TREATMENT OF ANGULAR DEPENDENCE AND SPATIAL DEPENDENCE OF ENERGY FLUX

The scattered angular energy flux, I^s, in (44) depends on Z, λ and ω . Refer to the Equation (44), we see that it involves ω and photon direction Ω' in the scattering in term, it is preferable to express these two direction variable by one only, ω . The relationship is expressed by Equation (39). Instead of cumbersome direct integration of equation (42), the solution for $I^s(Z, \lambda, \omega)$ has been represented by a power series of Legendre polynomial such that

$$I^S(Z, \lambda, \omega) = \sum_{m=0}^{\infty} A_m I_m(Z, \lambda) P_m(\omega) \qquad (44)$$

where $P_m(\omega) = a_{0,m} + a_{1,m}\,\omega + a_{2,m}\,\omega^2 + \ldots + a_{m,m}\,\omega^m \qquad (45)$

$$= \sum_{\ell=0}^{\infty} a_{\ell m}\,\omega^{\ell}$$

and A_m is an arbitrary constant chosen as to reduce the computation in the analytical treatment of the problem. The orthogonality property of the Legendre polynomial is

$$\int_{-1}^{+1} P_m(\omega)\, P_\ell(\omega)\, d\omega = \frac{2}{2\ell + 1} \qquad \text{if } m = \ell$$

$$= 0 \qquad \text{if } m \neq \ell$$

Normalization of the integral $\frac{2}{2\ell + 1}$ is arbitrary and has been chosen to avoid formidable algebra.

A concise and applied treatment of the subject can be obtained from the literature [7, 8, 9] .

Rewriting (44) by a power series

$$I^S(Z, \lambda, \omega) = \sum_{\ell=0}^{\infty} \frac{2\ell + 1}{4\pi} I_\ell^S(Z, \lambda)\, P_\ell(\omega) \qquad (46)$$

The coefficients I_ℓ^s in (46) is known as the ℓ^{th} angular moment of $P_\ell(\omega)$ and dependent of (Z, λ)[8].

Multiply (46) by $P_m(\omega)$ and integrating over all solid angle.

$$\int_{\text{all } \Omega} I^S(Z,\lambda,\omega) P_m(\omega) d\Omega = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} I_\ell^S(Z,\lambda) \int_0^{2\pi} d\varphi \int_{-1}^{+1} P_m(\omega) P_\ell(\omega) d\omega$$

$$= I_m^s \cdot (2\pi) \frac{2m + 1}{4\pi} \frac{2}{2m + 1}$$

$$= I_m^S(Z, \lambda)$$

Let's expand the differential equation (42) with the help of (46)

$$\sum_{0}^{\infty} \frac{2\ell+1}{4\pi} \left[P_\ell(\omega)\ \omega \frac{\partial I^S}{\partial Z}\ (Z,\ \lambda) + \mu(\lambda)\ I_\ell{}^S(Z,\ \lambda) P_\ell(\omega) \right]$$

$$= \sum_{\ell=0}^{\infty} \frac{2\ \ell\ +\ 1}{4\pi} \int_{4\pi} d\Omega' \int_{\lambda'=0}^{\lambda} \left[I_\ell{}^0(Z,\ \lambda') P_\ell(\omega') \right.$$

$$\left. + I_\ell{}^S(Z,\ \lambda') P_\ell(\omega') \right] \eta \frac{k(\lambda',\lambda)}{2\pi} \delta(1+\lambda'-\lambda-\cos\Psi) \Big] \qquad (47)$$

In short hand we could write (47) as

$$\sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} FL_\ell(Z,\lambda) P_\ell(\omega) + \sum_{\ell=0}^{\infty} RM_\ell(Z,\lambda) P_\ell(\omega)$$

$$= \sum_{\ell=0}^{\infty} \frac{2\ell\ +\ 1}{4\pi} SC_\ell(Z,\ \lambda)\ P_\ell(\omega) \qquad (48)$$

where FL is the flow term, RM is the photon removal term and SC_ℓ is the scattering integral term.

Since (48) is an algebraic equation, obviously, from the property of linear independence of an orthogonal function,

$$FL_\ell(Z,\lambda) + RM_\ell(Z,\lambda) = SC_\ell(Z,\lambda) \qquad \ell = 0,\ 1,\ 2,\ 3\ \ldots.$$

Later $\omega P_\ell(\omega)$ has to be expanded to use other orthogonality relations [9]

$\ell = 0$ $\qquad \omega P_0(\omega) = \omega$

$\ell = 1$ $\qquad \omega P_1(\omega) = \omega^2 = \frac{2}{3} P_2(\omega) + \frac{1}{3} P_0(\omega)$

$\because \quad P_2(\omega) = \frac{1}{2}(3\omega^2 - 1)$

$\ell = 2$ $\qquad \omega P_2(\omega) = \frac{1}{2}(3\omega^3 - \omega) = \frac{3}{5} P_3(\omega) + \frac{9}{10}\omega - \frac{1}{2}\omega$

$$= \frac{3}{5} P_3(\omega) + \frac{2}{5} P_1(\omega)$$

$\because$ Generalizing,

$$\omega P_\ell(\omega) = \frac{\ell + 1}{2\ell + 1} P_{\ell+1}(\omega) + \frac{\ell}{2\ell + 1} P_{\ell-1}(\omega) \qquad (49)$$

Substitute (49) in the first term of (47) on the LHS and operate

$$\int_{4\pi} d\Omega\, P_m = 2\pi \int_{-1}^{+1} P_m\, d\omega$$

$$FL_m(Z, \lambda) = 2\pi \int_{-1}^{+1} d\omega\, P_m(\omega) \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} \frac{\partial I_\ell^s}{\partial Z}(Z, \lambda)\, \omega P_\ell(\omega)$$

$$= 2\pi \int_{-1}^{+1} d\omega\, P_m(\omega) \sum_{\ell=0}^{\infty} \left[\frac{\ell+1}{2\ell+1} P_{\ell+1}(\omega) \frac{\partial I_\ell^s}{\partial Z} + \frac{\ell}{2\ell+1} P_{\ell-1}(\omega) \frac{\partial I_\ell^s}{\partial Z}\right. \qquad (50)$$

where $m = \ell + 1$ for the first term in Equation (50) on the R.H.S.,

and $m = \ell - 1$ for the second term on the R.H.S. in (50).

Applying the orthogonal property, the three terms in (48):

$$FL_m(Z,\lambda) = \frac{m}{2m+1} \frac{\partial I^s_{m-1}(Z,\lambda)}{\partial Z} + \frac{m+1}{2m+1} \frac{\partial I^s_{m+1}(Z,\lambda)}{\partial Z}$$

$$RM_m(Z, \lambda) = \mu\, I^s_\ell(Z, \lambda)$$

$$SC_\ell(Z, \lambda) = \int_{4\pi} d\Omega\, P_m(\omega)\, SC(Z, \lambda, \omega) \qquad (51)$$

We have five angular variables - θ, φ, θ', φ' and cos Ψ, among which cos Ψ is a function of others.

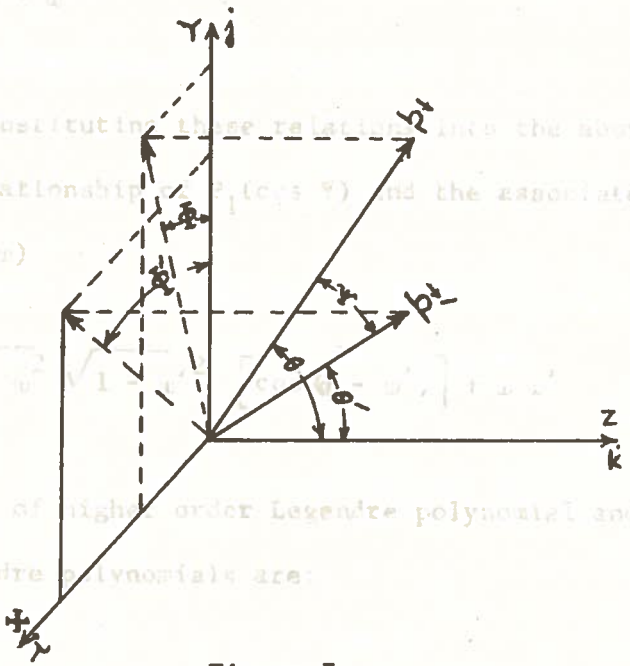

Figure 7

$$\Omega.\Omega' = \cos \Psi = (i \sin\theta \cos\varphi + j \sin\theta \sin\varphi + k \cos\theta)(i' \sin\theta$$

$$\cos\varphi' + j' \sin\theta' \sin\varphi' + k' \cos\theta')$$

$$= \sin\theta \sin\theta' \cos(\varphi - \varphi') + \cos\theta \cos\theta'$$

By definition:

$$\cos\theta = \omega$$

$$\sin\theta = \sqrt{1 - \omega^2}$$

$$\sin\theta' = \sqrt{1 - \omega'^2}$$

The result of substituting these relations into the above equation leads to the relationship of $P_1(\cos\Psi)$ and the associated Legendre polynomials $P_1'(\omega)$

$$\therefore \quad \cos\Psi = \sqrt{1 - \omega^2}\sqrt{1 - \omega'^2}\left[\cos(\varphi - \varphi')\right] + \omega\omega' \qquad (52)$$

The relationship of higher order Legendre polynomial and their associated Legendre polynomials are:

$$P_\ell(\cos \Psi) = P_\ell(\omega)P_\ell(\omega') + \sum_{m=1}^{\ell} \frac{(\ell-m)!}{(\ell+m)!} P_\ell^m(\omega') \cos m(\varphi'-\varphi) \qquad (53)$$

Integrating Equation (53) over the entire sphere:

$$\int P_\ell(\cos \Psi)\, d\Omega = 2\pi \int_{-1}^{1} P_\ell(\omega)\, P_\ell(\omega')\, d\omega' \qquad (54)$$

Since $\int_0^{2\pi} d\varphi' \cos m\,(\varphi' - \varphi) = 0.$

Excellent treatment of this subject on Associated Legendre polynomial (53) is available somewhere else [9].

Consider Equation (51). Recall $SC(Z, \lambda, \omega)$ which stands for the total scattering-in-term on the R.H.S. of (47) and consists of $SC^0 + SC^s$ where

SC^0 = uncollided photon from the source

SC^s = scattered photon from all solid angles.

When the Boltzmann's is operated by the operator $\int_\Omega P_m(\omega)\, d\Omega$ the non-vanishing term, SC_m^0, of the SC^0 part becomes:

$$\int_{-1}^{+1} d(\cos \Psi) \int_{\lambda'=0}^{\lambda'} d\lambda' \int_{4\pi} d\Omega' \left[2\pi\, P_m(\omega')P_m(\cos \Psi)\right] \frac{\lambda_0 e^{\frac{-\mu_0 Z}{\omega'}}}{4\pi\, |\omega'|}$$

$$H(\omega'\, Z)\, \frac{k(\lambda_0,\lambda)}{2\pi}\, \delta(1 + \lambda_0 - \lambda - \cos \Psi)$$

Integrating the $\int_{-1}^{1} d(\cos \Psi)$, we have

$$\int_{4\pi} d\Omega' \; P_m(\omega') P_m(1 + \lambda_0 - \lambda) \lambda_0 \frac{e^{-\frac{\mu_0 Z}{\omega'}}}{4\pi |\omega'|} H(\omega' Z) k(\lambda_0, \lambda) \qquad (55)$$

Similarly, operating $\int_{\Omega} P_m(\omega) d\Omega$ to the SC^S term,

$$\int_{-1}^{+1} d(\cos \Psi) \int_{-1}^{+1} d\omega' \; P_m(\omega') P_m(\cos \Psi) \int_0^{\lambda} d\lambda' \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} I_\ell^S(Z, \lambda')$$

$$P_\ell(\omega') \frac{k(\lambda', \lambda)}{2\pi} \delta(1 + \lambda' - \lambda - \cos \Psi) \int_0^{2\pi} d\varphi' \qquad (56)$$

Integrating over $\int_{-1}^{+1} d(\cos \Psi)$ with the Delta function, the non-vanishing term, SC_m^s part becomes:

$$\int_0^{\lambda} d\lambda' \; P_m(1 + \lambda' - \lambda) \; I_m^S(Z, \lambda') \; k(\lambda', \lambda) \qquad (57)$$

The source term also can be expanded as

$$S(Z, \omega, \lambda) = \sum_{\ell=0}^{\infty} \frac{2\ell + 1}{4\pi} S_\ell(Z, \lambda) P_\ell(\omega)$$

Operating by $\int_{\Omega} P_m(\omega) \; d\Omega$

$$2\pi \int_{-1}^{1} S(Z, \omega, \lambda) P_m(\omega) d\Omega = S_m(Z, \lambda) = S_m(\lambda) \; \delta(Z) \qquad (58)$$

where $S_m(\lambda) = \lambda_o \delta(\lambda - \lambda_0)$ for unscattered photons. Source term is zero for scattered part.

The Boltzman's equation for the scattered photons

$$SC_m(Z, \lambda) = SC_m^0(Z, \lambda) + SC_m^S(Z, \lambda) = FL_m^S(Z, \lambda) + RM_m^S(Z, \lambda) \quad (59)$$

where FL and RM are expressed in terms of I^S.

Treatment of Spatial Dependence

Rewriting (59) with index ℓ, we have

$$SC_\ell(Z, \lambda) = FL_\ell^S(Z, \lambda) + RM_\ell^S(Z, \lambda) \quad (60)$$

which means after operating $\int_{4\pi} P_\ell(\omega) d\Omega$ to Boltzman's equation, we have

$$\int_0^\lambda d\lambda' P_\ell(1 + \lambda' - \lambda) I_\ell^S(Z, \lambda') \, k(\lambda', \lambda)$$

$$+ \lambda_0 R(\lambda_0, \lambda) \, P_\ell(1 + \lambda_0 - \lambda) \int_0^{2\pi} d\varphi' \int_{-1}^{+1} d\omega' \frac{e^{-\frac{\mu_0 Z}{\omega'}}}{4\pi|\omega'|} P_\ell(\omega') H(\omega' Z)$$

$$= \frac{\ell + 1}{2\ell + 1} \frac{\partial I_{\ell+1}^S(Z,\lambda)}{\partial Z} + \frac{\ell}{2\ell + 1} \frac{\partial I_{\ell-1}^S(Z, \lambda)}{\partial Z}$$

$$+ \mu(\lambda) \, I_\ell^S(Z, \lambda) \quad (61)$$

Let us treat the space variable, Z, in the range $-\infty \leq Z \leq +\infty$ and express the dependence of $I_\ell^S(Z, \lambda)$ by an orthogonal set in powers of Z, say, Z^n. Precisely, this orthogonal set is never the Legendre Polynomials which has been used in the case of angular dependence.

Let the basic set of such a polynomial be

$1, Z, Z^2 \ldots Z^n$ and operate $I_\ell^S(Z, \lambda)$ by

$$\int_{-\infty}^{\infty} Z^n \, dZ \tag{62}$$

Making (62) dimensionless, we have

$$\int_{-\infty}^{\infty} \frac{\mu(E_0)^{n+1}}{n!} \, Z^n \, dZ \tag{63}$$

where $\mu(E_0) = \mu_0$

Operating (63) to (61) we will define a moment with a second index, n, such that it is independent of space or direction,

$$B_{n,\ell}(\lambda) = \frac{\mu_0^{n+1}}{n!} \int_{-\infty}^{\infty} Z^n I_\ell^S(Z, \lambda) dZ \tag{64}$$

Equation (64) is in essance obtained from $I^S(Z, \lambda, \omega)$ by operating with $\frac{\mu_0^{n+1}}{n!} \int_{-\infty}^{\infty} Z^n \, dZ \int_{4\pi} d\Omega \, P_\ell(\omega)$. This operator when operated on

total flux $I(Z, \lambda, \omega)$ yields the total moments, $bn\ell(\lambda)$, for the scattered and unscattered flux. Equation (66) is defined as the moments for the scattered angular energy flux $I_0^S(Z, \omega, \lambda)$

Consider the first term at the right hand side of Equation (61).

After operating by (63)

$$\frac{\mu_0^{n+1}}{n!} \frac{\ell + 1}{2\ell + 1} \int_{-\infty}^{\infty} Z^n \, dZ \frac{\partial I_{\ell+1}^S(Z, \lambda)}{\partial Z} \tag{65}$$

employing integration by parts, we have,

$$\frac{\mu_0^{n+1}}{n!} \frac{\ell+1}{2\ell+1} \left[Z^n I_{\ell+1}^S \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} I_\ell^S(Z,\lambda) n Z^{n-1} dZ \right] \tag{66}$$

Since $I_\ell^S(Z, \lambda)$ is an exponentially decaying function,

$$\lim_{|Z| \to \infty} Z^n I_{\ell+1}^S = 0$$

Equation (66) reduces to

$$- \frac{\ell + 1}{2\ell + 1} \mu_0 \left[\frac{\mu_0^n}{(n-1)!} \int_{-\infty}^{\infty} Z^{n-1} \, I_{\ell+1}^S \, (Z, \lambda) \, dZ \right] \tag{67}$$

By the definition of moments, it becomes

$$- \frac{\ell + 1}{2\ell + 1} \mu_0 \left[B_{n-1, \, \ell+1} \, (\lambda) \right] \quad n = 1, 2, 3, \ldots \tag{68}$$

Similarly, the second term at right hand side of (61) reduces to

$$-\frac{\ell}{2\ell+1}\,\mu_0\,B_{n-1,\,\ell-1}(\lambda) \qquad n = 0,\ 1,\ 2,\ 3\ \ldots \tag{69}$$

When the scattering-in term due to scattered photons of (61) is operated with (65), 23 have,

$$\int_0^{\lambda} d\lambda' P_\ell(1+\lambda'-\lambda) I_\ell^S(Z,\lambda') k(\lambda',\lambda) \left[\frac{\mu_0^{n+1}}{n!}\int_{-\infty}^{\infty}\right.$$

$$Z^n dZ I_\ell^S(Z,\lambda') = \int_0^{\lambda} d\lambda'\, B_{n,\ell}(\lambda') P_\ell(1+\lambda'-\lambda)\, k(\lambda',\lambda)$$

Similarly, the unscattered photon in the scattering-in term becomes

$$\lambda_0 k(\lambda_0,\lambda)\, P_\ell(1+\lambda_0-\lambda) \int_0^{2\pi} d\varphi' \int_{-1}^{+1} d\omega'\, P_\ell(\omega')$$

$$\frac{\mu_0^{n+1}}{n!}\int_{-\infty}^{\infty} e^{\frac{-\mu_0 Z}{\omega'}} Z^n H(\omega' Z)\, dZ \tag{70}$$

Defining the moments of unscattered photons

$$C_{n,\,\ell} = 2\pi \int_{-1}^{+1} d\omega' \frac{P_\ell(\omega')}{4\pi|\omega'|} \frac{\mu_0^{n+1}}{n!}\int_{-1}^{\infty} e^{\frac{-\mu_0 Z}{\omega'}} Z^n H(\omega' Z)\, dZ$$

Equation (70) can be written as

$$\lambda_0 k(\lambda_0, \lambda)\, P_\ell\, (1 + \lambda_0 - \lambda)\, C_{n,\,\ell} \tag{71}$$

The function $C_{n\ell}$* can also be expressed as

$$C_{n\ell} = \tfrac{1}{2} \int_{-1}^{+1} (\omega')^n\, P_\ell(\omega')\, d\omega' \tag{72}$$

where

$$P_0(\omega') = 1$$

$$P_1(\omega') = \omega$$

$$P_2(\omega') = \tfrac{1}{2}(3\,\omega^2 - 1)$$

$$P_3(\omega') = \tfrac{1}{2}(5\,\omega^3 - 3\omega)$$

$$P_4(\omega') = \frac{1}{8}(35\,\omega'^4 - 30\,\omega'^2 + 3)$$

$$P_5(\omega') = \frac{1}{8}(15\,\omega - 70\,\omega'^3 + 63\,\omega'^5)$$

$$P_6(\omega') = \frac{1}{16}(-5 + 105\,\omega'^2 - 315\,\omega'^4 + 231\,\omega'^6)$$

$$P_7(\omega') = \frac{1}{16}(-35\,\omega' + 315\,\omega'^3 - 693\,\omega'^5 + 429\,\omega'^6)$$

It is worth while to see that (72) bears the following property**:

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* Gamma Ray Transport Theory, pp. 23-24, unpublished notes, W. O. Doggett

** Unpublished notes, Wesley O. Doggett, North Carolina State Univ.

(1) when n is even and ℓ is odd, R.H.S. of (72) vanishes when integrated over $\int_{-1}^{+1} d\omega'$

(2) Similarly $C_{n\ell} = 0$ if n is odd and ℓ is even

(3) $(\omega')^n$ can be expressed as a linear combination of $P_m(\omega')$ with $m \leq n$; if $n < \ell$, $C_{n\ell}$ will vanish due to orthogonality relation.

The recursion relationship for $C_{n\ell}$ can be expressed*

$$C_{n\ell} = \frac{2^{\ell} n! \left(\frac{n+\ell}{2}\right)!}{\left(\frac{n-\ell}{2}\right)! \, (n+\ell+1)!} \quad \text{if } n-\ell \text{ is non-negative even integer} \qquad 72(a)$$

$$= 0 \quad \text{if } n-\ell \text{ is odd} \qquad 72(b)$$

$$= 0 \quad \text{if } n < \ell \qquad 72(c)$$

In particular,

$$C_{2n+1,\,0} = 0 \qquad 72(d)$$

$$C_{0,0} = 1 \qquad 72(e)$$

$$C_{2n,0} = \frac{1}{2n+1} \quad n = 1, 2, 3, \ldots \qquad 72(f)$$

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* Dr. W. O. Doggett - Gamma Ray Transport theory, unpublished notes, p. 23, Equation (97)

Now, when the last term of (61) is operated by (63), we have

$$\mu(\lambda)\, B_{n,\ell}\,(\lambda) \tag{73}$$

Rearranging (59) and substituting the simplified terms for S_{c_ℓ}, RM_ℓ^S and FL_ℓ^S, we have

$$RM_\ell^S = S_{c_\ell} - FL_\ell^S$$

$$\mu(\lambda)\, B_{n,\ell}(\lambda) = \int_{\lambda'=\lambda_0'}^{\lambda'} B_{n,\ell}(\lambda') P_\ell(1 + \lambda' - \lambda)\, k(\lambda', \lambda)\, d\lambda'$$

$$+ \lambda_0\, k(\lambda_0, \lambda)\, P_\ell(1 + \lambda_0 - \lambda)\, C_{n\ell}$$

$$+ \frac{\mu_0}{2\ell+1}\left[(\ell+1)\, B_{n-1,\ell+1}(\lambda) + \ell\, B_{n-1,\ell-1}(\lambda)\right] \tag{74}$$

where $n = 0, 1, 2, 3, \ldots$

and $B_{-1,\ell} = 0$ for ℓ either positive or negative.

Thus Equation (74) is the interlinked doubly index integral linear equation, in one variable, λ, for the angular and spatial moments for the scattered angular energy flux I_0^S in an infinite homogeneous medium for a plane isotropic, monoenergetic source emitting 1 photon/cm^2/sec.

CHAPTER 5

COMPUTATION OF THE MOMENTS AND RECONSTRUCTION OF ANGULAR ENERGY FLUX

It is necessary, for the subsequent derivation of the equations required in the computations of the moments, that we clarify

$$B_{n\ell}(\lambda) = 0 \quad \text{for} \quad (n - \ell) \text{ odd or negative}$$

for which $C_{n\ell} = 0$

Consider the Equation (74). Say, $n = 0$, $\ell = 1$, we have,

$$\mu B_{01}(\lambda) = \int_{\lambda'=\lambda'_0}^{\lambda'} B_{01}(\lambda') \, k(\lambda',\lambda) \, P_{\ell}(1 + \lambda' - \lambda) \, d\lambda' \qquad (75)$$

From 72(a) $C_{01} = 0$

From (74) conditions on $B_{-1,\,\ell}$

$$B_{-1,\,1} = 0$$

$$B_{-1,\,-1} = 0$$

The integral vanishes for $\lambda' = \lambda_0$

$$\therefore \quad B_{01}(\lambda_0) = 0$$

The general function for $B_{n\ell}(\lambda)$ can be expressed by a Taylor series expansion about λ_0 ;

$$B_{n,\ell}(\lambda) = B_{n,\ell}(\lambda_0) + \frac{\lambda-\lambda_0}{1!}\frac{dB_{n\ell}(\lambda_0)}{d\lambda} + \frac{(\lambda-\lambda_0)^2}{2!}\frac{d^2B_{n,\ell}}{d\lambda^2}(\lambda_0) + \cdots + \frac{d^mB_{n,\ell}}{d\lambda^m}(\lambda_0) \qquad (76)$$

If $B_{n\ell}(\lambda_0)$ and all its derivatives $\frac{d^mB_{n\ell}(\lambda_0)}{d\lambda^m}$ at λ_0 vanish identically, the functions vanishes for all λ. This is a property for Taylor series which is evident from (76) with respect to λ, and take

$\lambda' = '$, $n = 0$, $\ell = 1$

$$\mu\frac{dB_{01}(\lambda)}{d\lambda} + B_{01}(\lambda)\frac{d\mu}{d\lambda} = B_{01}(\lambda)k(\lambda,\lambda)P_1 + \int_{\lambda_0}^{\lambda} B_{01}(\lambda')\frac{d}{d\lambda}\left[k(\lambda',')P_\ell(1+\lambda'-\lambda)\right]d\lambda'$$

At $\lambda = \lambda_0$

$$\mu\frac{dB_{01}}{d\lambda}(\lambda) + 0 = 0 + 0$$

$$\mu \neq 0$$

$$\therefore \quad \frac{dB_{01}}{d\lambda}(\lambda) = 0$$

In general, it is possible to derive that,

$$\frac{d^m B_{01}(\lambda_0)}{d\lambda^m} = 0$$

Therefore in (76) $B_{01}(\lambda) = 0$. For $n = 1$, $\ell = 0$, we have from (74)

$$\mu(\lambda)B_{1,0}(\lambda) = \int_{\lambda'=\lambda_0}^{\lambda'} B_{1,0}(\lambda')P_0(1 + \lambda' - \lambda)\, k(\lambda',\lambda)d\lambda'$$

$$+ \lambda_0 k(\lambda_0,\lambda)P_0(1 + \lambda_0 - \lambda)\, C_{1,0}$$

$$+ \mu_0 \left[B_{0,1}(\lambda)\right] \qquad (77)$$

where $C_{1,0}$ and $B_{0,1}$ are zeros.

For $\lambda = \lambda_0$, the integral in (77) vanishes. Hence $B_{1,0}(\lambda_0) = 0$

also we can show $\dfrac{d^m B_{1,0}(\lambda_0)}{d\lambda^m} = 0.$

Therefore, in general, we can show

$$B_{n,\ell}(\lambda) = 0 \qquad n - \ell < 0 \qquad (78)$$

$$C_{n\ell} = 0 \quad \text{for } n - \ell = \text{odd}$$

A physical explanation to this kind of affair is very obvious from the fact that the term $C_{n,\ell}$ was generated for the uncollided flux which vanishes for $n - \ell$ being negative or odd. $B_{n,\ell}$ is an identical counterpart of $C_{n\ell}$ and has been generated for the scattered flux.*

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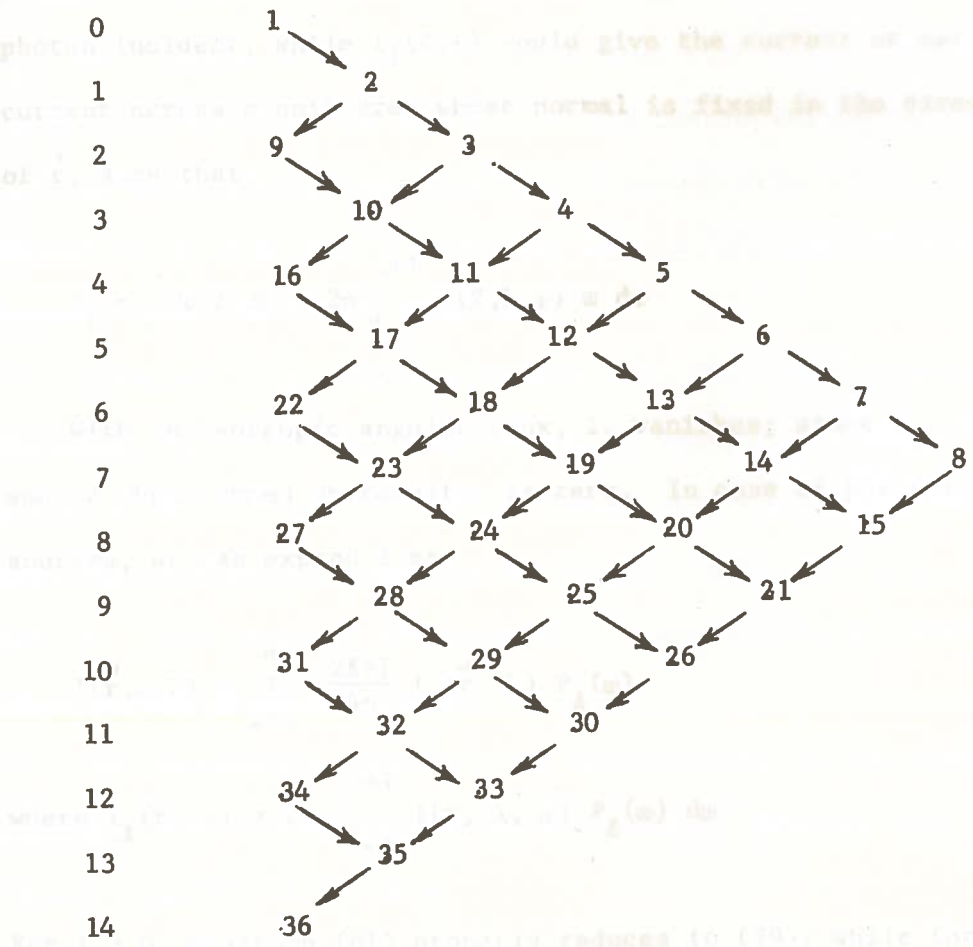
* W. O. Doggett - Unpublished notes, pp. 26, Gamma Ray Transport Theory, North Carolina State University, Raleigh, North Carolina.

The following table would show the general sequence of calculating the moments $B_{n,\ell}$ for an isotropic plane source:

TABLE



For $(n-\ell)$ odd or negative, $B_{n\ell}(\lambda)$ is zero, which has been shown by Equation (78). The integers represent the non-zero values of

$B_{n,\ell}(\lambda)$ for n = 0 to 14 and ℓ = 0 to 7 and sequence of number shows the scheme of computing the moments in IBM 360 computer.

For plane isotropic source $\ell = 0$. For a point source,

$$I_0(Z, \lambda) = \int_{\Omega} I(Z,\lambda,\Omega)\, d\Omega = 2\pi \int_{-1}^{+1} I(Z,\lambda,\omega)\, d\omega \qquad (79)$$

gives the energy flux at $\vec{r}$ summed over all the directions of the photon incident, while $I_1(Z,\lambda)$ would give the current or net energy current across a unit area whose normal is fixed in the direction of $\vec{r}$, such that,

$$I_1 = \int \omega\, I\, d\Omega = 2\pi \int_{-1}^{+1} I(Z,\lambda,\omega)\, \omega\, d\omega \qquad (80)$$

With an isotropic angular flux, I_1 vanishes; since I_1, in +Z and -Z when summed up results, is zero. In case of point isotropic sources, we can expand I as

$$I(\vec{r},\omega,\lambda) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} I_\ell(\vec{r}, \lambda)\, P_\ell(\omega) \qquad (81)$$

where $I_\ell(r, \lambda) = 2\pi \int_{-1}^{+1} I(r, \lambda, \omega)\, P_\ell(\omega)\, d\omega$

For $\ell = 0$, Equation (81) properly reduces to (79), while for $\ell = 1$, we obtain energy current density as shown in (80).

We are interested in calculating moments for $\ell = 0$ and $0 \leq n \leq N$, where N is a fixed integer such that a knowledge of moments $B_{n0}(n = 0, 1, 2, \ldots N)$ determines I_0 accurately. It is obvious from (74) that we cannot calculate $B_{4,0}$, say, until we have calculated $B_{3,1}$.

To calculate $B_{3,1}$ we must know $B_{2,2}$ and $B_{2,0}$; to calculate $B_{2,2}$, we have to have $B_{1,3}$ and $B_{1,1}$; to calculate $B_{2,0}$ we need to know $B_{1,1}$ and finally to calculate $B_{1,1}$ we must know $B_{0,0}$. Essentially we need to calculate the moments for any particular choice of (n,ℓ) set, we would move in the direction of arrows only. Thus for n = 0, 14 and $\ell = 0$, 7 or 14 x 7 matrix we need to calculate 36 non-zero moments along the 8 main diagonals.

Different source geometries have been discussed* elsewhere. Obviously for 14 x 7 $B_{n\ell}$ computation, we need $B_{n,0}(\lambda)$ where n = 0, 2, 4, 6, 8, 10, 12 and 14. Wilkins, Goldstein[3], et al, have been content with moments for $B_{n,0}(\lambda)$ where n = 0, 2, 4, 6, 8 and 10. Their work is obviously 10 x 7 calculation for moments as shown on the preceeding table.

For computation purposes only Equation (74) can be simplified further. For a fixed pair of values n and ℓ, Equation (74) is a linear equation of the form,

$$\mu(\lambda)v(\lambda) = \int_{\lambda_0}^{\lambda} H(\lambda,\lambda')\, v(\lambda')\, d\lambda' + T(\lambda) \qquad (82)$$

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* NYO 3075 Report. Equation 11(a) through 18(b). It is superfluous only to discuss them. An infinite plane angular source is given by

$$\frac{2m+1}{4\pi} P_m(\omega),\ m > 1$$

where $v(\lambda) = B_{n\ell}(\lambda)$ (83)

$$H(\lambda,\lambda') = k(\lambda', \lambda)\, P_\ell(1 + \lambda' - \lambda) \tag{84}$$

$$T(\lambda) = \frac{\mu_0}{2\ell+1} \left\{(\ell+1)\, B_{n-1,\ell+1}(\lambda) + \ell B_{n-1,\ell-1}(\lambda)\right.$$

$$\left. + \lambda_0\, k(\lambda_0,\lambda)\, P_\ell(1 + \lambda_0 - \lambda)\, C_{n,\ell}\right\} \tag{85}$$

The integral Equation (82) is of Volterra type* where the upper limit in the integral is the independent variable λ .

The value $v(\lambda_0)$ can be determined immediately, since, for $\lambda = \lambda_0$, the integral vanishes in (82)

$$\mu(\lambda_0)B_{n,\ell}(\lambda_0) = T(\lambda_0) = \frac{\mu_0}{2\ell+1} \left\{(\ell+1)B_{n-1,\ell+1}(\lambda_0) + \ell\, B_{n-1,\ell-1}(\lambda_0)\right\}$$

$$+ \lambda_0\, k(\lambda_0, \lambda_0)\, P_\ell(1)\, C_{n\ell}$$

From (33) $k(\lambda_0, \lambda_0) = \frac{3}{4}$

$$\therefore \quad B_{n\ell}(\lambda_0) = \frac{\ell+1}{2\ell+1} B_{n-1,\ell-1}(\lambda_0) + \frac{\ell}{2\ell+1} B_{n-1,\ell-1}(\lambda_0) + \frac{3}{4}\frac{\lambda_0}{\mu_0} C_{n\ell}$$

For $n = 0$, $\ell = 0$

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* NYO 3075 Report. p. 35.

$$B_{00}(\lambda_0) = \frac{3}{4}\frac{\lambda_0}{\mu_0} C_{00} = \frac{3}{4}\frac{\lambda_0}{\mu_0}$$

For $n = 1$, $\ell = 1$

$$B_{1,1}(\lambda_0) = \frac{2}{3} B_{0,2}(\lambda_0) + \frac{1}{3} B_{0,0}(\lambda_0) + \frac{3}{4}\frac{\lambda_0}{\mu_0} C_{1,1}$$

$B_{02} = 0$ from (78)

$C_{11} =$ from 72(a)

$\therefore$ $B_{11}(\lambda_0)$ can be calculated .

In general, at the source energy, E_0 or λ_0* ,

$$B_{n\ell}(\lambda_0) = \frac{3}{4}\frac{\lambda_0}{\mu_0} (n + 1) C_{n\ell} \qquad (86)$$

For plane isotropic case $\ell = 0$ and $C_{2n,0} = \frac{1}{2n + 1}$

$$B_{2n,0}(\lambda_0) = \frac{3}{4}\frac{\lambda_0}{\mu_0} (2n+1) \frac{1}{(2n+1)} = \frac{3}{4}\frac{\lambda_0}{\mu_0} \qquad (87)$$

To evaluate the integral $\int_{\lambda_0}^{\lambda} H(\lambda,\lambda') v(\lambda') d\lambda'$ in (82) numerical integration has been performed

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* NYO 3075, Equation (20), p. (35).

Let $\lambda_i = \lambda_0 + i(\Delta\lambda)$, $i = 1, 2, \ldots$

By trapizoidal rule, from (82)

$$\mu(\lambda_1)v(\lambda_1) = \tfrac{1}{2}[H(\lambda_1,\lambda_0)\, v(\lambda_0) + H(\lambda_1,\lambda_1)\, v(\lambda_1)]\, \Delta\lambda + T(\lambda_1) \quad (88)$$

$$H(\lambda_1,\lambda_1) = k(\lambda_1,\lambda_1)\, P_\ell(1) = \frac{3}{4} \quad (\because P_\ell(1) = 1)$$

$$v(\lambda_1) = \frac{T(\lambda_1) + H(\lambda_1,\lambda_0)\, \dfrac{v(\lambda_0)}{2}\, \Delta\lambda}{\mu(\lambda_1) - \dfrac{3}{8}\Delta\lambda} \quad (89)$$

For $i \geq 2$

$$\mu(\lambda_T)\, v(\lambda_i) = \int_{\lambda_0}^{\lambda} H(\lambda,\lambda')v(\lambda')\, d\lambda' + T(\lambda)$$

$$- \sum_{k=0}^{i-1} (\lambda_i,\lambda_k)\, v(\lambda_k)\, M_{ik}$$

$$+ H(\lambda_i,\lambda_i)\, v(\lambda_i)\, M_{ii} + T(\lambda_i) \quad (90)$$

where M_{ik} and M_{ii} are the coefficients depending upon the integration scheme, either Simpson's or Trapizoidal rule or any other existing method. For Simpson's rule,

$$M_{ii} = \frac{\Delta\lambda}{3}$$

$$H(\lambda_i,\lambda_i) = \frac{3}{4}$$

So (90) can be simplified to

$$v(\lambda_i) = \frac{T(\lambda_i) + \sum_{k=0}^{i-1} H(\lambda_i,\lambda_k)\, v(\lambda_k)\, M_{ik}}{\mu(\lambda_i) - \frac{\Delta\lambda}{4}} \qquad (91)$$

Equations (86), (87), (89) and (91) together with 72(a), 72(b), 72(c), 72(d), 72(e) and 72(f) make the complete set for computation purposes.

Computational Procedure with the help of IBM 360 Mixed Mode Digital Computer.

Choice of $\Delta\lambda$ in the Equation (89) and (91) is critical and interpolation of $\mu(\lambda_1)$ is necessary between the range $\lambda_0 \leq \lambda \leq \lambda_i$. It has been suggested for high energy, such as 10 Mev $\Delta\lambda = 0.01$ and for low energy, such as 0.50 Mev $\Delta\lambda = 0.08$ is adequate. This choice is governed by the factors that numerical errors are to be minimized and the denominators in (89) and (91) does not come out negative. We have used $\Delta\lambda = 0.04$ and kept it constant. To initialize the computation, we have calculated $v(\lambda_1)$ by trapizoidal rule and $v(\lambda_i)$, $i \geq 2$ by Simpson's Rule. Equation (86) computes $v(\lambda_0)$ and used it in (89). For $n = 0$, $\ell = 0$, from (84), $T(\lambda_1) = \lambda_0\, k(\lambda_0, \lambda_1)\, P_\ell(1 + \lambda_0 - \lambda_1)\, C_{n\ell}$. In the computer program, function subprogram TLMDA, has it as Z1 = LMDAO*H*Y

where

LMDAO = λ_0

$H = HLMDA(KNK,P) = H(\lambda,\lambda_0)\ P_\ell(1 + \lambda_0 - \lambda)$

$KNK = KLNSH(A,B) = K(\lambda_0,\lambda)$

$P = PLEGD = P_\ell(1 + \lambda_0 - \lambda)$

$Y = C_{n\ell}$

The function KLNSH computes $KNK = K(\lambda_0,\lambda)$ and PLEGD computes $P_\ell(1 + \lambda_0 - \lambda)$. Function C(N,L) computes C_{oo} . So $v(\lambda_1)$ is known for $n = 0$, $\ell = 0$ $v(\lambda_0)$ is calculated by the Function BNLO. $v(\lambda_1)$ is calculated by the Function BNL1.

To compute $v(\lambda_2)$ we need to know $T(\lambda_2)$ and $\mu(\lambda_2)\ T(\lambda_2)$ requires $B_{-1,1}$ and $B_{-1,-1}$ which are obviously zero for reasons discussed in 72(a) through 72(f) and (74). As a matter of fact, for $n = 0$ and $\ell = 0$, all $T(\lambda_i) = \lambda_0\ k(\lambda_0,\lambda_1\ P_\ell(1 + \lambda_0 - \lambda_1) = Z1 =$ LMDAO*H*Y. We must distinguish here that i in

$$\lambda_i = \lambda_0 + i(\Delta\lambda)$$

does not bear any relationship of n in $B_{n\ell}$ or $C_{n\ell}$; (i) in λ_i would mean the degradation of energy by interaction processes and in a priori decision to make how far we are interested in following up the energy starting λ_0 to any arbitrary $\lambda_i = \lambda_0 + i(\Delta\lambda)$,

and represents the number of interval chosen for integration purposes only, where n and ℓ in $B_{n\ell}$ or $C_{n\ell}$ is the index arising out of the polynomial expansion schemes.

$\mu(\lambda_2)$ is calculated by the interpolation subroutine INTERP*[10]. It is a 3 point interpolation formula and an error estimate has been given somewhere else[11].

We chose to calculate the moments for $\lambda_0 \leq \lambda \leq \lambda_0 + 6.0$ (where 6 represents 3 compton scattering ranges). For 1 Mev source strength and $\Delta\lambda = 0.04$, we would need,

$$\lambda = \lambda_0 + i\,\Delta\lambda$$

$$6 + \lambda_0 = \lambda_0 + i(\Delta\lambda)$$

or,

$$i = \frac{6.0}{.04} = 150 \text{ intervals for integration or } 150 + 1 = 151$$

points. For 3 Mev source, water medium, we chose $\Delta\lambda = .02$, so we calculated 351 moments corresponding to each set of n and ℓ. Computation up to $\lambda_0 \leq \lambda \leq \lambda_0 + 2$ is straight forward but Klein-Nishina Kernal vanishes for $\lambda_i > \lambda_0 + 2.0$; following figure would clarify the scheme of computation:

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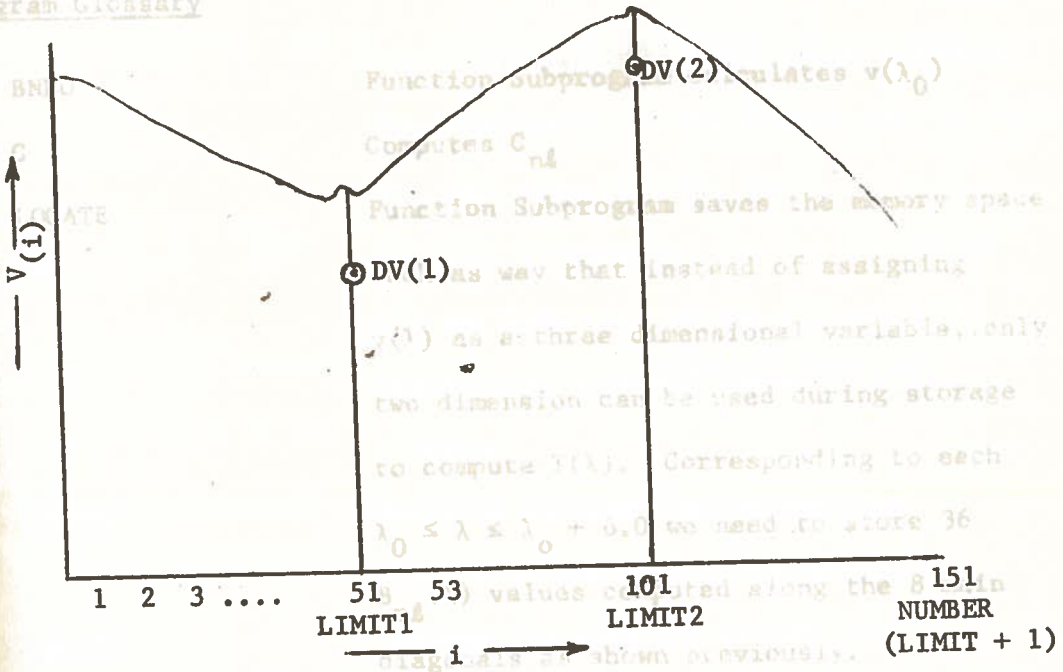
* I am indebted to Dr. Myron H. Young for letting me use this subroutine package which is based on divided difference method. Theoretical treatment of the subject could be found in page 38, Reference (10).



At i = 51, we computed the v(51) and also calculated the product of H_i v(i) starting i = 2 to 51 and set $T(\lambda_i) = 0$ since $k(\lambda,\lambda) = 0$. Use of subprogram VALUE1 with the subroutine INT gives Dv(1) - designated as the moment at the discontinuous point (dP); to compute v(52) we integrated over the region $H_i v(3)$ to $H_i v(51)$ and the H_{Dv} Dv(1); to compute v(53), integrate i = 4 to i = 51, integration over i = 51 to i_{dP} is 0. We used VALUE1 and for i_{dP} to i = 52, we used VALUE2. The process is repetitive over the region i = 101 to i = 151 where VALUE1 and VALUE2 function subprogram has been used in conjunction with INT. The program contains 2 subroutines and 14 subprograms and have been listed below according to the order they are called by the main program.

Program Glossary

1) BNLO — Function Subprogram calculates $v(\lambda_0)$

2) C — Computes $C_{n\ell}$

3) LOCATE — Function Subprogram saves the memory space such as way that instead of assigning $y(\lambda)$ as a three dimensional variable, only two dimension can be used during storage to compute $T(\lambda)$. Corresponding to each $\lambda_0 \leq \lambda \leq \lambda_o + 6.0$ we need to store 36 $B_{n\ell}(\lambda)$ values computed along the 8 main diagonals as shown previously.

4) KLNSH — Computes Klein-Nishina Kernal, $K(\lambda_0,\lambda)$

5) PLEGD — Computes $P_\ell(1 + \lambda_0 - \lambda)$

6) HLMDA — Computes product of $K(\lambda_0,\lambda)P_\ell(1 + \lambda_0 - \lambda)$

7) TLMDA — Computes $T(\lambda)$

8) INTERP — Subroutine computes $\mu(\lambda)$, the absorption coefficient in TU/elec.

9) BNL1 — Function computes $v(\lambda_1)$

10) FAC — Function subprogram calculates $C_{n\ell}$ and is called by the subprogram C(N,L) to compute the factorial value of $C_{n\ell}$ where $(n - \ell)$ is a non-negative even integer.

11) INTG — Function subprogram generates the product of $H(\lambda',\lambda) \times V(I)$ for $1 \leq I \leq 151$ and

	calls the subroutine INT, which performs the integration of the product $H(\lambda',\lambda)*v(I)$ and INTG compute v(I), for $3 \leq I \leq 51$.
12) INT	Performs integration for $3 \leq I \leq 151$ or 351 for both even or odd intervals by trapizoidal rule and Simpson's rule combined.
13) VALUE1	Performs the same job as INTG for $2 \leq I \leq 51$ for $v(2) \leq v(I) \leq v(51)$
14) VALUE2	Perform the same job as INTG for $51 \leq I \leq 101$
15) VALUE3	Performs the same job as VALUE 1 for $52 \leq I \leq 101$
16) PLEGUR	Computes the reconstruction of flux, $I^S, e^{\mu_o r}_{4\pi r^2}$.

Transforming Moments for Plane Isotropic Source to Point Isotropic Source.

A theoretical proof is available* for the relationship between

$$B^{\text{Plane}}_{n\ell}(\lambda) = C_{n\ell}\, B^{\text{Point}}_{n\ell}(\lambda) \qquad (92)$$

We are content with the application of (92). A simplified treatment is given** in Reference (8)

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* NYO 3075. pp 32-34

** W.O. Doggett unpublished notes, Gamma Ray Transport Theory, p. 33-37.

$C_{n\ell}$ for $\ell = 0$, and $(n - \ell)$ being non-negative even integer, is given by

$$C_{2n,0} = \frac{1}{2n+1} \qquad n = 0, 1, 2 \tag{93}$$

To compute $B_{n\ell}^{\text{Point}}(\lambda)$ we multiply the computed moments for plane isotropic source by $(2n + 1)$ for a particular set of $(n, 0)$ and this is done during the reconstruction of angular energy flux.

Reconstruction of Angular Energy Flux

$$(Q = 4\pi r^2 I_0^S(\lambda) e^{\mu_0 r}) \tag{94}$$

we are interested in calculating the scattered energy flux, $I^S(\lambda)$ in order to calculate the dose buildup factor (B_r) or energy build-up factor (BE) and total dose. The dose buildup factor is used in various shield design problems and health physics.

Angular energy flux is defined as,

$$I(Z, \lambda, \omega) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} I_\ell(Z, \lambda) P_\ell(\omega) \tag{95}$$

For dose buildup factor we need,

$$\int_{4\pi} I(Z, \lambda, \omega) = d\Omega = I_0(Z, \lambda)$$

$$I_0(\lambda) = \int_0^\infty I_0(Z, \lambda)\, dZ$$

Consider a point source. Some clue as to the spatial behavior of $I_0^s(r, \lambda)$ may be obtained by recalling the unscattered flux,

$I_0^0(r, \lambda) = \lambda_0^S(\lambda - \lambda_0) \frac{e^{-\mu_0 r}}{4\pi r^2}$. $4\pi r^2 I_0^0(r, \lambda)$ behaves spatially like $e^{-\mu_0 r}$ for $r \geq 0$. We except that $I_0^s(r, \lambda)$, the scattered component will behave the same way. Let us propose a series representation for $4\pi r^2 I_0^s(r, \lambda)$ as

$$4\pi r^2 I_0^s(r,\lambda) = e^{-\mu_0 r} \sum_{n=0}^{\infty} u_n(\lambda)\, u_n(\mu_0 r)$$

$$= e^{-\mu_0 r} \sum_{n=0}^{\infty} u_n(\lambda)\, u_n(y) \qquad (96)$$

where $y = \mu_0 r$ and $u_n(y)$ is a polynomial which can be expressed as a linear combination.

$$u_n(y) = a_{0n} + a_{1n} y + a_{2n} y^2 + \ldots + a_{nn} y^n \qquad (97)$$

The coefficient of $u_n(y)$ is independent of r and been made dimensionless $(\mu_0 r)$. The coefficient of $u_n(\lambda)$ depends on λ. L.V. Spencer and U. Fano[12] have discussed in a great detail on the

theoretical treatment of the reconstruction of angular energy flux. W. O. Doggett[8] has discussed this topic in his notes. We should be content with the principle involved only.

Like the Legendre polynomial, in Equation (45), to find the coefficients in (97), let us multiply (97) by another polynomial $U^+_m(\mu_0 r)$ and integrate,

$$\int_0^\infty U^+_m(\mu_0 r)\, 4\pi r^2 I_0{}^S(r, \lambda) d(\mu_0 r) = \sum_{n=0}^{\infty} u_n(\lambda) \int_0^\infty e^{-\mu_0 r} u_n(\mu_0 r)$$

$$U^+_m(\mu_0 r)\, d(\mu_0 r) \qquad (98)$$

If $U(\mu_0 r)$ and $U^+_m(\mu_0 r)$ are orthogonal with weight factor $e^{-\mu_0 r}$ so that,

$$\int_0^\infty e^{-\mu_0 r} u_n(\mu_0 r)\, U^+_m(\mu_0 r) = \delta_{nm} = 1 \text{ for } n = m$$

$$= 0 \text{ for } n \neq m \qquad (98')$$

then $U^+_m(\mu_0 r)$ and $U_m(\mu_0 r)$ are said to form an orthonormal set.

R.H.S. of (98) = $u_m(\lambda)$

$$\therefore\ u_m(\lambda) = \int_0^\infty U^+_m(\mu_0 r)\, 4\pi r^2\, I_0{}^S(r, \lambda)\, d(\mu_0 r) \qquad (99)$$

R.H.S. of (99) bears a similarily with the definition of moment

for point source. Thus $U^{+}_{m}(\mu_0 r)$ is taken to have the form of a linear combination of even powers of $(\mu_0 r)$

$$U^{+}_{m}(\mu_0 r) = a^{+}_{0,m} + a^{+}_{2,m} y^2 + a^{+}_{4,m} y^4 + \ldots + a^{+}_{2m,m} y^{2m} \qquad (100)$$

and

$$U_m(\lambda) = a^{+}_{0,m} B^{Pt.}_{0,0}(\lambda) + a^{+}_{2,m}(2!) B^{Pt.}_{2,0}(\lambda) + \\ + a^{+}_{2m,m}(2m)! B^{Pt.}_{2m,0}(\lambda) \qquad (101)$$

Equations (97) and (100) for U_n and U^{+}_{m} together with (98') and the arbitrary assignment of $a^{+}_{0,m} = 4$ complete specify the two sets of functions.

$U_n(\mu_0 r)$ and $U^{+}_{m}(\mu_0 r)$ are the set of biorthonormal function and the derivation for it can be referred to (8). We are content in writing the set as follows:

$U_n(y)$ and $U^{+}(y)$ for even angular indices

$V_n(y)$ and $V^{+}_{n}(y)$ for odd angular indices

However for the moments for a plate source we need only $U_n(y)$ and $U^{+}_{m}(y)$* where

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* Refer to page 45, "Double P_m solutions to the Gamma-Ray Transport Equation by the Moments Method", - M.H. Young, Ph.D. Dissertation.

$$U_0(y) = 1 \qquad 102(a)$$

$$U_1(y) = \tfrac{1}{2}(1 - y) \qquad 102(b)$$

$$U_2(y) = 1/8(3 - 5y + y^2) \qquad 102(c)$$

$$U_3(y) = 1/48(15 - 33y + 12y^2 - y^3) \qquad 102(d)$$

$$U_4(y) = 1/384(105 - 279y + 141y^2 - 22y^3 + y^4) \qquad 102(e)$$

$$U_5(y) = 1/3840(945 - 289ty + 1830y^2 - 405y^3 + 35y^4 - y^5) \qquad 102(f)$$

$$U_6(y) = 1/46080(10395 - 35685y + 26685y^2 - 7500y^3 + 930y^4 - 51y^5 + y^6) \qquad 102(g)$$

$$U_7(y) = 1/645120(135135 - 509985y + 435960y^2 - 146685y^3 + 23310y^4 - 1848y^5 + 70y^6 + y^7) \qquad 102(h)$$

In general form, Equation (100) can be written as

$$U^+_m(\mu_0 r) = \sum_{i=1}^{m} a^+_{wi,m} y^{2i} \qquad (103)$$

Comparing (103) with the set of Equation 102(a) through 102(h),

we can write in general

$$a^{+}_{2i,m} = (-1)^{i} \binom{m}{i} \frac{1}{(2i)!} \tag{104}$$

with $\binom{m}{i}$ is the binomial coefficient;

$$\binom{m}{i} = \frac{m!}{i!(m-i)!}$$

Substitute (104) into (101)

$$u_m(\lambda) = \sum_{i=1}^{m} a^{+}_{2i,m} (2i)! \; B^{Pt}_{2i,0}(\lambda) \tag{105}$$

Substitute (104) into (105)

$$u_m(\lambda) = \sum_{i=1}^{m} (-1)^{i} \binom{m}{i} B^{Pt}_{2i,0}(\lambda) \tag{106}$$

Substitute (106) in (96) with change of subscript n to m and instead of considering an infinite series, fewer moments, say N + 1 are calculated for the series, for convergence,

$$4\pi r^2 \, I_0{}^{S}(r, \lambda) = e^{-\mu_0 r} \sum_{n=0}^{N} u_n(\lambda) \, U_n(\mu_0 r) \tag{107}$$

Equation (106) gives $u_n(\lambda)$ e.g., $u_0(\lambda)$, $u_1(\lambda)$, $u_2(\lambda)$. We have finally

$$4\pi r^2 I_0^S(r,\lambda) = e^{-\mu_0 r} \sum_{n=0}^{N} \left\{ \sum_{i=0}^{n} (-1)^i \binom{m}{i} B_{2i,0}^{Pt}(\lambda) \right\} u_n(\mu_0 r) \quad (108)$$

Interchanging the summations, (108) can be written as,

$$4\pi r^2 I_0^S(r,\lambda) = e^{-\mu_0 r} \sum_{i=0}^{N} \left\{ \sum_{n=i}^{N} (-1)^i \binom{n}{i} U_n(\mu_0 r) \right\} B_{2i,0}^{Pt}(\lambda)$$

$$Q_{2i}^{N}(\mu_0 r) = \sum_{n=i}^{N} (-1)^i \binom{n}{i} U_n(\mu_0 r)$$

In our computation, differential angular energy spectra means

$$4\pi r^2 I_0^S(r,\lambda,N)\, e^{\mu_0 r} = \sum_{n=0}^{N} (2n+1)\, B_{2n,0}^{P\ell}(\lambda) Q_{2n}^{N}(\mu_0 r) = Q \quad (109)$$

Calculation of dose and dose buildup factor:

Dose is computed as

$$D(r) = \int_E \mu\, a^{air}(E) I_0(r,E)\, dE \left[\text{cm}\right]^{-1} \left[\frac{\text{Mev. \# Photon}}{(\text{Mev})(\text{cm}^2)(\text{sec})} \right] [\text{Mev}]$$

or $\dfrac{\text{Mev}}{(\text{cm}^3)(\text{sec})}$ (110)

By definition for a point, monoenergetic source*, normalized to one photon of energy E_0 emitted per unit time:

$$I_0^0(r, E) = \frac{E_0 e^{-\mu_0 r}}{4\pi r^2} \delta(E - E_0) \quad (111)$$

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* Ref. to Equation 11(a), page 18, NYO 3075

$$\therefore\ D(r) = \mu_a{}^{air}(E_0)E_0 \frac{e^{-\mu_0 r}}{4\pi r^2} + \int \mu_a{}^{air}(E) I_0{}^{S}(r, E)dE \qquad (112)$$

$$= \left\{\begin{matrix}\text{Dose due to uncollided photon} \\ \text{flux } (D_0{}^{0})\end{matrix}\right\} + \left\{\begin{matrix}\text{Dose due to scattered} \\ \text{photon flux } (D_0{}^{S})\end{matrix}\right\}$$

The dose buildup factor is given by

$$B_r(r) = \frac{D_0{}^{0} + D_0{}^{S}}{D_0{}^{0}} = 1 + \frac{D_0{}^{S}}{D_0{}^{0}}$$

$$= 1 + \frac{\int \mu_a{}^{air}(E) I_0{}^{S}(r, E)dE}{\mu_a{}^{air}(E_0)E_0 \frac{e^{-\mu_0 r}}{4\pi r^2}}$$

or

$$B_r(r) = 1 + \frac{\int \mu_a{}^{air}(e) I_0{}^{S}(r,E) e^{\mu_0 r}\, 4\pi r^2}{\mu_a{}^{air}(E_0)E_0} \qquad (113)$$

From Equations (109) and (112), we have

$$B_r(r) = 1 + \frac{\int_E \mu_a{}^{air} Q}{\mu_a{}^{air}(E_0)E_0} dE$$

$$= 1 + \frac{\int \mu_a{}^{air}(\lambda) Q \frac{(0.511)}{\lambda^2} d\lambda}{\mu_0{}^{air}(E_0)E_0} \qquad (114)$$

In the computer program, the second term in the neumerator, on the R.H.S. of (114) has been calculated as BNL3(I).

To calculate dose $D_r(r)$ in (112), multiply the dose buildup factor, $B_r(r)$ by $\mu_0^{air}(E_0)E_0$. In the computer program, BNL4 corresponds to $\mu_0^{air}(E_0)E_0$. In the results, we have dividied the dose by $(e^{\mu_0 r})(4\pi r^2)$ as shown in Table 9.

Difference between $B_E(r)$ and $B_r(r)$

It is worthwhile to distinguish between the energy buildup factor, $B_E(E)$ and dose buildup factor. Energy buildup factor is defined as

$$B_E(Z) = \frac{\int I_0^0(Z,E)\,dE + \int I_0^S(Z,E)\,dE}{\int I_0^0} = 1 + \frac{\int I_0^S(Z,\lambda)dE}{\int I_0^0(Z,\lambda)dE}$$

From (41'),

$$B_E(Z) = 1 + \frac{\int \frac{I_0^S(Z,\lambda)d\lambda}{\lambda^2}}{\frac{E_0E_1(Z)}{2}} = 1 + \frac{2\lambda_0}{E_1(Z)\lambda_0}\int_{\lambda_0}^{\lambda}\left[\frac{I_0^S(Z,\lambda)}{\lambda^2}\right]d\lambda \quad (115)$$

CHAPTER 6

DISCUSSION OF RESULTS

A computer program has been developed to compute 14 x 7 moments for the plane isotropic source, differential angular energy spectra $\left[4\pi r^2 e^{\mu_0 r} I_0^S(r, \lambda)\right]$, total dose and the dose buildup factor $B_r(r)$ for point isotropic source. Table 1 gives the partial list of moments at 1 Mev for water.

Differential angular energy spectral $\left[4\pi r^2 e^{\mu_0 r} I_0^S(r,\lambda) = Q\right]$ has been computed for water, aluminum, concrete and iron at 1 Mev, for point isotropic source (Table 3-7). Calculation has been repeated for 3 Mev source for water medium. The results show excellent agreement at $\mu_0 r$ = 1, 2, 4, and 7 during the energy degradation process at most of the points. We considered the range of energy degradation, $\lambda_0 \leq \lambda \leq \lambda_0 + 6.0$. The computed value for Q at $\mu_0 r$ = 10, 15 and 20 significantly differs from the calculations for 10 x 7 moments[3]. In all these computations seven significant digits have been carried and to compute the absorption coefficient data at any energy, divided difference method of interpolation has been used. Except concrete, the absorption coefficient (μ) data (TU/electron) has been quoted from the report by Goldstein and Wilkins [3] in their report NBS 3075.

The fact that dose buildup factor $B_r(r)$ agrees fairly well with those reported, suggests that convergence of the moments method

is doubtful for 10 x 7 moments at the depth of penetration, greater than 7. It is possible that numerical error in the interpolation of μ, integration technique and approximation of moments with Legurra polynomial to compute I_0^S have contributed to the breakdown of the procedure at higher depth of penetration. No error analysis has been made; although in the subroutine INTERP used for interpolation of μ had the scope for the estimation of error. The subroutine INT uses a constant $\Delta\lambda = 0.04$ unlike Goldstein and Wilkins[3] at 1 Mev and at $\Delta\lambda = 0.02$ at 3 Mev. The author has made a subroutine for numerical integration by Richardson's deferred approach to a limit where an estimate for the error during the integration loop can be obtained. During the time this subroutine package was made, any change in the computer program, MOMENT, warranted difficulty in incorporating it and fitting it with the rest of the subroutine and function subprogram.

Total dose (Dr) and dose buildup factor $B_r(r)$ has been reported in the Table 8. Table 2 shows computed value for the Q_n^{2n} values as required during the reconstruction of angular energy flux at the different depth of penetrations.

Differential angular energy spectra, Q, has been plotted against λ at the different depth of penetration. The computer plotting program has been shown in Appendix 2.

To illustrate the method of computation in securing the data for the absorption coefficient, μ, (TU/electron) for a

homogeneous mixture, concrete was picked up as the representative of the group.

Absorption coefficient (μ) for a homogeneous mixture in TU/electron is given by*

$$\mu = \sum_i \beta_i \mu_i$$ where μ_i is the corresponding absorption coefficient for the i^{th} element and β_i is the electron fraction (116)

$$\beta_i = \frac{\frac{W_i Z_i}{A_i}}{\sum_i^J \frac{W_i Z_i}{A_i}}$$ where W_i is the weight fraction, Z_i is the atomic number and A_i is the atomic weight for the i^{th} element (117)

The composition of concrete is available in (13) as reported in the supplement to N.B.S. circular 583 (1959).

The absorption coefficient data for concrete has been reported in cm^2/gm [22]. The relationship between the unit for μ in (cm^2/gm) and TU/electron has been expressed as,

$$\mu, \text{ in units of } cm^2/gm = 0.4005 \frac{Z}{A} \mu(\text{TU/electron})$$ (118)

μ_i for each constituent in concrete was computed by (118) - the

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* NYO 3075, page 52

data for μ in units of cm^2/gm for each constituent at 1 Mev were collected from N.B.S. 1003 report [22]. β_1 was calculated by (117) and finally μ was calculated by (116). Appendix III shows the computer program and the elemental composition of concrete.

Total dose and scattered dose calculated as $Mev/cm^3 sec$ can be used to compute Mev/cm^2 sec. at 1 cm. which can be finally converted to 1r/hr at 1 cm* .

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* Rockwell-Reactor Shielding Design Manual, pp. 20, Figure 2.2.

TABLE 1

Partial List of Moments for Water Medium
Plane Isotropic:
Incident Gamma Energy: 1 Mev

E(Mev)	μ	λ	n	1	Moments
1.000	.3174	0.511	0	0	1.207466
0.9274	0.3283	0.551	0	0	1.144783
0.8646	0.3402	0.591	0	0	1.028009
0.8098	0.3519	0.631	0	0	0.987698
0.2068	0.6055	2.471	0	0	1.735726
0.2035	0.6089	2.511	0	0	1.779278
Moment at the discontinuous point					1.501615
0.1143	0.7267	4.471	0	0	2.143132
0.1133	0.7285	4.511	0	0	2.193506
Moment at the discontinuous point					2.241312
0.1000	0.7535	5.111	0	0	4.328661
0.0853	0.7866	5.9910	0	0	4.032865
0.0785	0.8008	6.511	0	0	4.159392
1.0000	0.3174	0.511	1	1	0.8049776
0.2068	0.6055	2.471	1	1	0.1380801
0.2035	0.6089	2.511	1	1	1.201722
Moment at the discontinuous point					-0.05721054
0.1143	0.7267	4.471	1	1	0.4003132
0.1133	0.7285	4.511	1	1	0.3930956
Moment at the discontinuous point					0.08766788
0.0785	0.8008	6.511	1	1	0.5590915

TABLE 2

Polynomials for point isotropic source.

$$4\pi r^2 e^{\mu_o r} I_0^5(r, \lambda) \approx \sum_{n=0}^{N} B2n, 0(\lambda)\, Q_{2n}^N(\mu_o r), \text{ where } Q_{2n}^N(\mu_o r) =$$

$(-1)^n \sum_{J=n}^{N} (J_n)\, U_J(\mu_o r)$ and U_J represents the coefficients of Laguerre polynomials (Biorthonormal)

$\mu_o r$	Q_0^7	Q_2^7	Q_4^7	Q_6^7	Q_8^7	Q_{10}^7	Q_{12}^7	Q_{14}^7
1	0.248013	3.24583	-6.470504	7.729380	5.95540	2.892967	-0.808311	0.099275
2	-0.307783	2.836534	-2.625293	1.556586	1.556586	-0.535862	-0.061426	-0.005737
4	0.162542	-1.666048	6.137010	-6.423772	4.111815	-1.680175	0.403570	-0.043494
7	-0.176606	1.688476	-6.551561	12.059784	-9.082679	3.911522	-0.950954	0.102018
10	0.064601	-1.005045	5.477050	-1.066873	19.007972	10.886229	3.150769	-0.381766
15	-0.481305	4.965816	-21.527328	50.463780	-67.490204	48.706040	-15.585933	1.949079
20	-1.450383	6.732107	-3.414770	-38.860244	107.480646	-123.781721	64.896087	-10.601802

TABLE 3

Differential Angular Energy Spectra,

$$4\pi r^2 e^{\mu_o r} I_o S(r, \lambda) \quad (Q)$$

Medium: Water
Incident Gamma Energy: 1 Mev
Source: Point Isotropie

E (Mev)		$\mu_o r$ = 1		2		4		7	
1.000	0.511	1.2075	(1.21)	2.4149	(2.41)	4.8299	(4.82)	8.4523	(8.44)
0.8646	0.591	1.0152	(1.04)	2.0566	(2.18)	4.2147	(4.71)	7.6300	(9.13)
0.7616	0.671	0.9151	(0.942)	1.9216	(2.04)	4.1654	(4.66)	7.9819	(9.63)
0.6804	0.751	0.8501	(0.876)	1.8389	(1.95)	4.1486	(4.68)	8.2112	(10.10)
0.5609	0.911	0.7858	(0.813)	1.7749	(1.90)	4.2077	(4.83)	8.5980	(10.90)
0.4771	1.07	0.7747	(0.805)	1.7899	(1.93)	4.3302	(5.04)	8.9252	(11.50)
0.4440	1.151	0.7886		1.8384		4.4802		9.2617	
0.4020	1.271	0.8123		1.8886		4.5798		9.4364	
0.3674	1.391	0.8554		1.9744		4.7446		9.7220	
0.3295	1.551	0.9275	(0.967)	2.0955	(2.28)	4.9390	(5.84)	10.0445	(13.30)
0.2853	1.791	1.0750		2.3317		5.3271		10.7403	
0.2619	1.951	1.1929		2.5132		5.6330		11.3091	
0.2421	2.110	1.3187	(1.45)	2.7011	(3.10)	5.9605	(7.45)	11.9240	(16.50)
0.2211	2.311	1.4857		2.9576		6.4572		12.8661	
0.2035	2.511	1.6496	(1.76)	3.2244	(3.61)	7.0419	(8.57)	13.9314	(18.90)
0.1972	2.591	1.3785	(1.47)	3.1087	(3.40)	7.2210	(8.49)	14.6051	(19.0)

TABLE 3, Continued

E (Mev)		$\mu_o r$ = 1		2		4		7	
0.1805	2.831	1.4933	(1.49)	3.5469	(3.53)	8.5221	(8.91)	17.3868	(20.1)
0.1622	3.150	1.6030	(1.56)	3.9847	(3.77)	9.8686	(9.58)	10.2951	(21.6)
0.1489	3.432	1.6811		4.2945		10.8330		22.3999	
0.1392	3.671	1.7387		4.5165		11.5221		23.9900	
0.1280	3.992	1.7985		4.7623		12.3062		25.6439	
0.1174	4.353	1.7591	(1.89)	4.7999	(4.83)	12.6188	(12.8)	26.4125	(29.1)
0.1133	4.57	1.7415		4.8269		12.7803		26.7979	
0.1103	4.633	1.7955		5.2340		14.2691		30.1702	
0.1049	4.871	2.8402		9.4035		27.5807		59.6552	
0.1008	5.069	2.8888		9.9396		29.8467		65.1398	
0.0934	5.471	2.6985		9.7858		30.2907		66.8680	
0.0895	5.709	2.5550	(2.05)	9.5170	(5.67)	29.8926	(15.8)	66.3504	(36.8)
0.0847	6.033	2.3882		9.1487		29.1696		65.1178	
0.0810	6.309	2.2588		8.7706		28.1769		63.1048	
0.0785	6.51	2.4231		9.2969		29.7448		66.5892	

TABLE 3, Continued

E (Mev)	$\mu_o r$ = 10		$\mu_o r$ = 15		$\mu_o r$ = 20	
1.000	12.0746	(12.1)	18.1118	(18.10)	24.1482	(24.1)
0.8646	11.2446	(14.3)	17.6812	(24.4)	24.5988	(36.3)
0.7616	12.1972	(15.7)	19.8562	(28.1)	28.1237	(43.1)
0.6804	12.7477	(16.8)	20.9745	(30.8)	29.8090	(47.7)
0.5609	13.5081	(18.6)	22.3665	(34.5)	31.8418	(54.2)
0.4771	14.0555	(19.8)	23.3116	(37.1)	33.2125	(58.8)
0.4440	14.6023		24.2454		34.5574	
0.4020	14.8792		24.7519		35.3450	
0.3674	15.3010		25.4033		36.2114	
0.3295	15.7862	(22.9)	26.1929	(43.0)	37.3506	(68.1)
0.2853	16.8441		27.9273		39.8538	
0.2619	17.7076		29.3482		41.8965	
0.2421	18.6374	(28.4)	30.8812	(53.4)	44.0784	(81.6)
0.2211	20.0874		33.2925		47.5271	
0.2035	21.7297	(32.4)	35.9156	(60.8)	51.1655	(95.70)
0.1972	22.9794	(32.8)	38.2253	(61.9)	54.6431	(96.70)
0.1805	27.4614	(34.7)	45.7912	(65.6)	65.4592	(103.0)
0.1622	32.1489	(37.2)	53.7344	(70.5)	76.8071	(112.0)
0.1489	35.5510		59.5146		85.0725	
0.1392	37.9933		63.6665		91.0200	
0.1280	40.7986		68.4417		97.8538	
0.1174	42.0704	(50.0)	70.6332	(95.0)	100.9788	(156.0)

TABLE 3, Continued

E (Mev)	$\mu_o r$ = 10		15		20	
0.1133	42.7008		71.7027		102.5080	
0.1103	48.1897		81.0770		115.9094	
0.1049	95.7664		161.8184		231.3547	
0.1008	104.7563		172.2780		253.5586	
0.0934	107.7691		182.7459		261.4605	
0.0895	107.0473	(63.40)	181.6899	(121.0)	260.0042	(206.0)
0.0847	105.1814		178.6865		255.8013	
0.0810	101.9969		173.3711		248.2253	
0.0785	107.6200		182.9089		261.9085	

TABLE 4

Differential Angular Energy Flux

$$4\pi r^2 e^{\mu_o r} I_o S(r, \lambda) \quad (Q)$$

Medium: Water
Incident Gamma Energy: 3 Mev
Source: Point Isotropie

E (Mev)		$\mu_o r$ = 1		2		4		7	
3.000	0.1703	0.7178	(0.734)	1.4356	(1.47)	2.8712	(2.94)	5.0247	(5.14)
2.4292	0.2104	0.5715	(0.602)	1.1288	(1.22)	2.2065	(2.51)	3.7506	(4.54)
2.0411	0.2504	0.5028	(0.528)	1.0071	(1.08)	2.0118	(2.26)	3.4934	(4.15)
1.5468	0.3304	0.4395	(0.458)	0.9015	(0.954)	1.8514	(2.02)	3.2647	(3.57)
1.0421	0.4903	0.4066	(0.428)	0.8458	(0.902)	1.7446	(1.93)	3.0419	(3.56)
0.7857	0.6504	0.4188	(0.442)	0.8706	(0.929)	1.7767	(1.96)	3.0506	(3.57)
0.5739	0.8904	0.4630	(0.409)	0.9426	(1.01)	1.8679	(2.08)	3.1448	(3.71)
0.4222	1.2103	0.4017	(0.579)	0.9223	(1.16)	1.9353	(2.31)	3.2893	(4.04)
0.3675	1.3905	0.4222		0.9708		2.0381		3.4651	
0.3429	1.4902	0.4390		1.0026		2.0950		3.5553	
0.3213	1.5904	0.4570		1.0382		2.1649		3.6773	
0.2953	1.7304	0.4906		1.1010		2.2808		3.8676	
0.2762	1.8501	0.5213	(0.811)	1.1576	(1.53)	2.3872	(2.94)	4.0469	(5.06)
0.2542	2.0102	0.5687		1.2455		2.5544		4.3284	
0.2354	2.1708	0.6225	(0.962)	1.3455	(1.77)	2.7478	(3.39)	4.6529	(5.84)
0.2271	2.2501	0.6372	(0.868)	1.3975	(1.70)	2.8702	(3.33)	4.8595	(5.78)

TABLE 4, Continued

E (Mev)		$\mu_o r$ = 1		2		4		7	
0.2193	2.3301	0.6611	(0.902)	1.4641	(1.76)	3.0179	(3.39)	5.1050	(5.86)
0.2036	2.5098	0.7079		1.5974		3.3173		5.6016	
0.1885	2.7109	0.7468		1.7160		3.5922		6.0603	
0.1768	2.8902	0.7720	(0.985)	1.8010	(1.96)	3.7976	(3.76)	6.4058	(6.47)
0.1602	3.1898	0.8011		1.9121		4.0799		6.8853	
0.1498	3.4112	0.8339		2.0028		4.2880		7.2352	
0.1400	3.6500	0.8669		2.0898		4.4847		7.5669	
0.1320	3.8712	0.8926		2.1579		4.6401		7.8306	
0.1281	3.9891	0.8961		2.1703		4.6729		7.8879	
0.1225	4.1714	0.8961		2.1800		4.7055		7.9476	
0.1175	4.3489	1.2764		2.8479		5.8908		9.9298	
0.1133	4.5101	1.3534		3.0307		6.2796		10.5807	
0.1118	4.5707	1.3864	(1.29)	3.1126	(2.68)	6.4573	(5.28)	10.8782	(9.13)
0.1099	4.6497	1.4003		3.1515		6.5455		11.0251	
0.1076	4.7491	1.4027		3.1660		6.5842		11.0881	
0.1000	5.1100	1.3966		3.1833		6.6489		11.1913	
0.0948	5.3903	1.3669		3.1371		6.5716		11.0588	
0.0911	5.6092	1.3636		3.1453		6.6029		11.1119	
0.0870	5.8736	1.3587		3.1500		6.6274		11.1551	
0.0856	5.9696	1.3859		3.2166		6.7714		11.4009	
0.0828	6.1703	1.4466		3.3622		7.0833		11.9328	

TABLE 4, Continued

E (Mev)	$\mu_o r$ = 10		$\mu_o r$ = 15		$\mu_o r$ = 20	
3.0000	7.1780	(7.34)	10.7673	(11.0)	14.3553	(44.7)
2.4292	5.2324	(6.67)	7.6195	(10.4)	9.9563	(14.2)
2.0411	4.9392	(6.13)	7.2884	(9.56)	9.6015	(13.2)
1.5468	4.6299	(5.54)	6.8198	(8.59)	8.9581	(12.0)
1.0421	4.2683	(5.19)	6.2256	(7.95)	8.1341	(11.4)
0.7857	4.2494	(5.16)	6.1675	(7.87)	8.0363	(11.4)
0.5739	4.3546	(5.34)	6.2936	(8.13)	8.1963	(11.5)
0.4222	4.5945	(5.81)	6.6644	(8.82)	8.6927	(11.7)
0.3675	4.8461		7.0439		9.2066	
0.3429	4.9646		7.2033		9.3996	
0.3213	5.1396		7.4721		9.7677	
0.2953	5.3971		7.8344		10.2277	
0.2762	5.6452	(7.29)	8.1955	(11.0)	10.7022	(13.7)
0.2542	6.0350		8.7637		11.4446	
0.2354	6.4853		9.4180		12.2975	
0.2271	6.7722	(8.34)	9.8294	(12.7)	12.8298	(15.8)
0.2193	7.1114	(8.65)	10.3144	(12.8)	13.4554	(15.6)
0.2036	7.8004		11.3022		14.7304	
0.1885	8.4397		12.2209		15.9151	
0.1768	8.9234	(9.54)	12.9175	(14.10)	16.8120	(17.1)
0.1602	9.5972		13.8897		18.0613	
0.1498	10.0839		14.5916		18.9641	

TABLE 4, Continued

E (Mev)	$\mu_o r$ = 10		15		20	
0.1400	10.5454		15.2570		19.8224	
0.1320	10.9130		15.7874		20.5069	
0.1281	10.9936		15.9047		20.6556	
0.1225	11.0797		16.0296		20.8174	
0.1175	13.8148		20.0155		26.0565	
0.1133	14.7177		21.3170		27.7435	
0.1118	15.1304	(13.40)	21.9119	(19.90)	28.5137	(25.0)
0.1099	15.3337		22.2041		28.8868	
0.1000	15.5630		22.5206		29.2836	
0.0948	15.3785		22.2484		28.9213	
0.0911	15.4531		22.3547		29.0534	
0.0870	15.5160		22.4447		29.1684	
0.0856	15.8589		22.9447		29.8138	
0.0828	16.6038		24.0258		31.2287	

TABLE 5

Differential Angular Energy Spectra,

$$4\pi r^2 e^{\mu_o r} I_o^S (r, \lambda) \quad (Q)$$

Medium: Concrete
Incident Gamma Energy: 1 Mev
Source: Point Isotropie

E (Mev)		$\mu_o r$ = 1	2	4	7
1.0000	0.511	1.2063	2.4127	4.8253	8.4443
0.8646	0.591	1.0120	2.0462	4.1796	7.5394
0.7616	0.671	0.9155	1.9250	4.1826	8.0427
0.6804	0.751	0.8362	1.7861	3.9541	7.6882
0.5609	0.911	0.7683	1.7033	3.9237	7.7925
0.4771	1.071	0.7770	1.7937	4.3054	8.7474
0.4440	1.151	0.7799	1.8005	4.3128	8.7382
0.3782	1.351	0.8367	1.9295	4.6066	9.3244
0.3295	1.551	0.9190	2.0608	4.7903	9.5763
0.2853	1.791	1.0672	2.2987	5.1810	10.2716
0.2516	2.031	1.2546	2.5920	5.6924	11.2273
0.2250	2.271	1.4095	2.7896	6.0001	11.7553
0.2035	2.511	1.5978	3.0869	6.6302	12.8879
0.1972	2.571	1.3324	2.9689	6.7810	13.4797
0.1755	2.912	1.4795	3.5140	8.3633	16.7830
0.1601	3.192	1.5276	3.7309	9.0390	18.2126

TABLE 5, Continued

E (Mev)		$\mu_o r$			
		1	2	4	7
0.1472	3.471	1.5890	3.9614	9.7265	19.6647
0.1407	3.632	1.6203	4.0767	10.0686	20.3890
0.1334	3.831	1.6528	4.2001	10.4402	21.1787
0.1268	4.030	1.6710	4.2882	10.7264	21.7961
0.1219	4.192	1.5735	4.0343	10.0792	20.4687
0.1153	4.432	1.4686	3.8481	9.7105	19.7530
0.1133	4.511	1.4721	3.8690	9.7710	19.8790
0.1094	4.671	1.6036	4.4939	11.75909	24.1122
0.1041	4.908	2.2811	6.9082	18.8665	39.0807
0.1000	5.110	2.1891	6.7436	18.6006	38.6371
0.0941	5.430	2.0484	6.4944	18.2022	37.9752
0.0895	5.709	1.8331	5.8818	16.5867	34.6553
0.0870	5.874	1.7197	5.5482	15.6872	32.7958
0.0842	6.069	1.5633	5.0849	14.4197	30.1658
0.0815	6.270	1.4892	4.8323	13.6997	28.6584
0.0785	6.511	1.4558	4.6868	13.2381	27.6708

TABLE 5, Continued

E (Mev)	$\mu_o r$		
	10	15	20
1.000	12.0632	18.0948	24.1254
0.8646	11.0880	17.4237	24.2995
0.7616	12.3341	20.2134	28.8529
0.6804	11.8212	19.3490	27.5722
0.5609	12.0455	19.7384	28.1205
0.4771	13.6247	22.4160	31.9737
0.4440	13.5966	22.3620	31.8937
0.3782	14.5411	24.0112	34.3523
0.3295	14.8720	24.4536	34.8936
0.2853	15.9212	26.1616	37.3621
0.2516	17.3660	28.5303	40.7625
0.2250	18.1310	29.7609	42.4924
0.2035	19.8658	32.5427	46.3997
0.1972	20.9642	34.5742	49.4814
0.1755	26.1991	43.2823	61.8848
0.1601	28.4768	47.0929	67,3115
0.1472	30.7860	50.9573	72.8235
0.1407	31.9380	52.8860	75.5747
0.1334	33.1955	54.9926	78.5828
0.1268	34.1795	56.6444	80.9370
0.1219	32.0929	53.1803	75.9868

TABLE 5, Continued

E (Mev)	$\mu_o r$		
	10	15	20
0.1153	30.9884	51.3615	73.3762
0.1133	31.1866	51.6872	73.8469
0.1094	37.9242	62.9542	89.8931
0.1041	61.6111	102.4583	146.1907
0.1000	60.9422	101.3994	144.6541
0.0941	59.9467	99.8224	142.3643
0.0895	54.7238	91.1489	129.9890
0.0870	51.7933	86.2804	123.0339
0.0842	47.6483	79.3837	113.1936
0.0815	45.2658	75.4156	107.5264
0.0785	43.6992	72.7904	103.8059

TABLE 6

Differential Angular Energy Spectra,

$$4\pi r^2 e^{\mu_o r} I_0^S (r, \lambda) \qquad (Q)$$

Medium: Aluminum
Incident Gamma Energy: 1 Mev
Source: Point Isotropie

E (Mev)		$\mu_o r$ = 1		2		4	6	7	
1.000	0.511	1.2078	(1.21)	2.4156	(2.42)	4.8312	(4.84)	8.4546	(8.46)
0.8646	0.591	1.0153	(1.05)	2.0566	(2.19)	4.2137	(4.72)	7.6254	(9.17)
0.7616	0.671	0.9153	(0.943)	1.9217	(2.04)	4.1644	(4.66)	7.9768	(9.60)
0.6804	0.751	0.8510	(0.878)	1.8419	(1.96)	4.1590	(4.68)	8.2385	(10.0)
0.5609	0.911	0.7848	(0.813)	1.7705	(1.90)	4.1901	(4.80)	8.5478	(10.7)
0.4771	1.071	0.7737		1.7849		4.3087		8.8616	
0.4440	1.151	0.7874	(0.817)	1.8330	(1.96)	4.4565	(5.11)	9.1905	(11.6)
0.3782	1.351	0.8361		1.9306		4.6395		9.4992	
0.3295	1.551	0.9256	(0.968)	2.0873	(2.27)	4.9056	(5.79)	9.9482	(13.0)
0.2853	1.791	1.0704		2.3160		5.2727		10.5970	
0.2516	2.031	1.2443	(1.30)	2.5744	(2.84)	5.6966	(6.84)	11.3751	(15.1)
0.2250	2.271	1.4338	(1.50)	2.8557	(3.16)	6.2095	(7.49)	12.3292	(16.4)
0.2035	2.511	1.6213	(1.73)	3.1522	(3.52)	6.8439	(8.28)	13.4821	(18.1)
0.1972	2.591	1.3511	(1.44)	3.0271	(3.31)	6.9847	(8.18)	14.0658	(18.4)
0.1755	2.912	1.4852	(1.47)	3.5390	(3.45)	8.4942	(8.63)	17.2668	(19.3)
0.1601	3.192	1.5578		3.8341		9.3974		19.1990	

TABLE 6, Continued

E (Mev)		$\mu_o r$ 1		2		4		7	
0.1472	3.471	1.6208		4.0764		10.1340		20.7832	
0.1407	3.632	1.6437	(1.58)	4.1664	(3.80)	10.4094	(9.53)	21.3773	(21.1)
0.1334	3.831	1.6734		4.2810		10.7581		22.1293	
0.1268	4.030	1.6739		4.3121		10.8829		22.4107	
0.1219	4.192	1.6522	(1.70)	4.2908	(4.16)	10.8797	(10.6)	22.4268	(23.7)
0.1153	4.432	1.5570		4.1435		10.6308		21.9629	
0.1133	4.511	1.5618		4.1686		10.7043		22.1203	
0.1094	4.671	1.7126		4.8962		13.0651		27.2473	
0.1041	4.908	2.4703	(1.63)	7.6891	(4.18)	21.5139	(11.0)	45.4219	(24.8)
0.1000	5.110	2.3684		7.5058		21.2255		44.9588	
0.0941	5.430	2.2140		7.2350		20.8121		44.3032	
0.0895	5.709	2.0655	(1.54)	6.8772	(4.03)	19.9813	(10.7)	42.6550	(24.2)
0.0870	5.874	1.9679		6.6394		19.4211		41.5370	
0.0842	6.069	1.8253		6.2209		18.2914		39.1795	
0.0815	6.270	1.7768		6.0729		17.8855		38.3345	
0.0785	6.511	1.7866	(1.35)	6.0613	(3.59)	17.7963	(9.63)	38.1242	(21.9)

TABLE 6, Continued

E (Mev)	$\mu_o r$ = 10		$\mu_o r$ = 15		$\mu_o r$ = 20	
1.0000	12.0778	(12.1)	18.1168	(18.1)	24.1547	(24.2)
0.8646	11.2338	(14.3)	17.6545	(24.5)	24.5478	(36.5)
0.7616	12.1847	(15.6)	19.8230	(27.9)	28.0595	(42.7)
0.6804	12.7948	(16.7)	21.0545	(30.5)	29.9165	(47.1)
0.5609	13.4154	(18.3)	22.1898	(33.8)	31.5671	(52.9)
0.4771	13.9362		23.0828		32.8560	
0.4440	14.4683	(19.9)	23.9872	(37.1)	34.1575	(58.9)
0.3782	14.9414		24.8052		35.3809	
0.3295	15.6074	(22.3)	25.8508	(41.8)	36.8254	(65.9)
0.2853	16.5877		27.4512		39.1335	
0.2516	17.7574	(25.9)	29.3689	(48.2)	41.8789	(73.5)
0.2250	19.2001	(28.1)	31.7314	(52.3)	45.2168	(79.7)
0.2035	20.9794	(30.8)	34.5976	(57.3)	49.2292	(89.7)
0.1972	22.0781	(31.1)	36.6460	(58.2)	52.3222	(90.4)
0.1755	27.2175	(33.0)	45.2973	(62.0)	64.6646	(96.8)
0.1601	30.3245		50.5460		72.1555	
0.1472	32.8741	(36.3)	54.8621	(68.2)	78.3149	(107.)
0.1407	33.8327		56.4856		80.6428	
0.1334	35.0433		58.5384		83.5710	
0.1268	35.5023		59.3218		84.6917	

TABLE 6, Continued

E (Mev)	$\mu_o r$ = 10		$\mu_o r$ = 15		$\mu_o r$ = 20	
0.1219	35.5365	(40.5)	59.3953	(46.1)	84.7805	(122.)
0.1153	34.8264		58.2239		83.1069	
0.1133	35.0745		58.6379		83.6929	
0.1094	43.3297		72.6021		103.5960	
0.1041	72.4306	(42.4)	121.6585	(80.0)	173.5169	(131.)
0.1000	71.7335		120.5686		171.9382	
0.0941	70.7553		119.0426		169.7473	
0.0895	68.1601	(41.4)	114.7402	(78.3)	163.6013	(129.)
0.0870	66.4007		111.8134		159.4360	
0.0842	62.6486		105.5322		150.4670	
0.0815	61.3049		103.2799		147.2605	
0.0785	60.9613	(37.4)	102.6852	(70.9)	146.4309	(118.)

TABLE 7

Differential Angular Energy Spectra,

$$4\pi r^2 e^{\mu_o r} I_o S(r, \lambda) \quad (Q)$$

Medium: Iron
Incident Gamma Energy: 1 Mev
Source: Point Isotropie

E		$\mu_o r$ = 1		2		4		7	
(Mev)									
1.0000	0.511	1.2031	(1.20)	2.4062	(2.40)	4.8124	(4.81)	8.4217	(8.41)
0.8646	0.591	1.0099	(1.04)	2.0426	(2.17)	4.1733	(4.69)	7.5216	(9.09)
0.7616	0.671	0.9122	(0.938)	1.9157	(2.03)	4.1519	(4.64)	7.9479	(9.58)
0.6149	0.831	0.8024		1.7714		4.0913		8.2139	
0.5156	0.991	0.7650	(0.791)	1.7382	(1.86)	4.1280	(4.72)	8.3941	(10.60)
0.4291	1.191	0.7762		1.7810		4.2438		8.6007	
0.3782	1.351	0.8081		1.8246		4.2634		8.5350	
0.3474	1.470	0.8512	(0.886)	1.8912	(2.04)	4.3469	(5.07)	8.6287	(11.20)
0.3133	1.631	0.9250		1.9982		4.4788		8.8047	
0.2853	1.790	0.9894	(1.03)	2.0559	(2.24)	4.4714	(5.29)	8.7107	(11.40)
0.2619	1.951	1.0693		2.1452		4.5526		8.8116	
0.2421	2.110	1.1379	(1.18)	2.2060	(2.42)	4.5871	(5.47)	8.8303	(11.60)
0.2211	2.311	1.2204		2.2858		4.6840		8.9659	
0.2035	2.511	1.2825	(1.35)	2.3441	(2.58)	4.7906	(5.70)	9.0980	(11.90)
0.1972	2.591	1.0271	(1.08)	2.1454	(2.33)	4.6343	(5.41)	9.0139	(11.50)
0.1805	2.831	1.0289	(0.998)	2.1940	(2.18)	4.7854	(5.06)	9.3143	(10.80)

TABLE 7, Continued

E (Mev)		$\mu_o r$							
		1		2		4		7	
0.1708	2.992	1.0160		2.1777		4.7561		9.2474	
0.1622	3.150	0.9768	(0.908)	2.0906	(1.98)	4.5552	(4.56)	8.8452	(9.69)
0.1507	3.391	0.9437		2.0138		4.3697		8.4681	
0.1407	3.632	0.8484		1.789		3.8503		7.4506	
0.1334	3.831	0.8018		1.6799		3.6005		6.9614	
0.1293	3.952	0.7384	(0.658)	1.5392	(1.40)	3.2908	(3.17)	6.3617	(6.68)
0.1243	4.111	0.6559		1.3628		2.9113		5.6291	
0.1196	4.273	0.5965		1.2457		2.6697		5.1661	
0.1133	4.510	0.4549		0.9676		2.0940		4.0652	
0.1076	4.749	0.3685		0.7906		1.7085		3.3114	
0.1016	5.030	0.3300	(0.289)	0.7133	(0.627)	1.5484	(1.44)	3.0064	(3.05)
0.0969	5.273	0.2794		0.6037		1.3104		2.5443	
0.0927	5.512	0.2311		0.4992		1.0835		2.1037	
0.0876	5.833	0.1724		0.3723		0.8077		1.5683	
0.0836	6.112	0.1285		0.2772		0.6013		1.1674	
0.0810	6.309	0.0995		0.2146		0.4655		0.9036	
0.0785	6.510	0.0821		0.1770		0.3837		0.7447	

TABLE 7, Continued

E (Mev)		$\mu_o r$ = 10		$\mu_o r$ = 15		$\mu_o r$ = 20	
1.000	0.511	12.0308	(12.0)	18.0465	(18.0)	24.0610	(24.0)
0.8646	0.591	11.0379	(14.2)	17.2395	(24.3)	23.8308	(36.1)
0.7616	0.671	12.1241	(15.6)	19.6606	(28.0)	27.7151	(42.8)
0.6149	0.831	12.7997		21.0318		29.7607	
0.5156	0.991	13.1160	(18.0)	21.5638	(33.2)	30.5165	(52.3)
0.4291	1.191	13.4189		22.0787		31.2913	
0.3782	1.351	13.2652		21.7757		30.8350	
0.3474	1.470	13.3709	(18.9)	21.8889	(34.9)	30.9391	(54.9)
0.3133	1.631	13.6073		22.2418		31.4460	
0.2853	1.790	13.4197	(19.2)	21.9016	(35.3)	30.9739	(53.8)
0.2619	1.951	13.5374		22.0705		31.2175	
0.2421	2.110	13.5297	(19.4)	22.0386	(35.6)	31.1592	(52.8)
0.2211	2.311	13.7138		22.3302		31.5709	
0.2035	2.511	13.8982	(19.6)	22.5493	(36.3)	31.8059	(55.8)
0.1972	2.591	13.9044	(19.4)	22.7308	(35.6)	32.2140	(53.4)
0.1805	2.831	14.3866	(18.2)	23.5236	(33.6)	33.3353	(50.1)
0.1708	2.992	14.2828		23.3515		33.0862	
0.1622	3.150	13.6579	(16.3)	22.3258	(30.1)	31.6337	(44.5)
0.1507	3.391	13.0714		21.3608		30.2725	
0.1407	3.632	11.4921		18.7762		26.6123	
0.1334	3.831	10.7338		17.5352		24.8572	
0.1293	3.952	9.8063	(14.2)	16.0193	(20.6)	22.7088	(30.1)
0.1243	4.111	8.6757		14.1738		20.0885	

TABLE 7, Continued

E (Mev)		$\mu_o r$ = 10		$\mu_o r$ = 15		$\mu_o r$ = 20	
0.1196	4.273	7.9653		13.0143		18.4464	
0.1133	4.510	6.2750		9,6835		14.5430	
0.1076	4.749	5.1103		8.3578		11.8547	(16.2)
0.1016	5.030	4.6445	(5.15)	7.5937	(9.46)	10.7713	(13.9)
0.0969	5.273	3.9305		6.4264		9.1156	(6.23)
0.0927	5.512	3.2498		5.3134		7.5367	
0.0876	5.833	2.4225		3.9609		5.6183	(13.1)
0.0836	6.112	1.8033		2.9483		4.1821	
0.0810	6.309	1.3958		2.2820		3.2369	
0.0785	6.510	1.1501		1.8805		2.6668	

TABLE 8

Dose Build Up Factor (Br)
Point Isotropie Source

Medium	Incident Gamma Energy	Relaxation length ($\mu_o r$)							
		1		2		4		7	
Water	1	1.9896	(2.13)	3.2727	(3.71)	6.3896	(7.68)	11.8785	(16.2)
Water	3	1.6010	(1.69)*	2.2457	(2.42)	3.5366	(3.91)	5.4095	(6.23)
Aluminum	1	1.9699	(2.02)	3.1842	(3.31)	6.0769	(6.57)	11.1372	(13.1)
Concrete	1	1.9580		3.1382		5.9111		10.6992	
Iron	1	1.8429	(1.87)	2.7892	(2.89)	4.9452	(5.39)	8.6680	(10.2)

TABLE 8, Continued

Medium	Incident Gamma Energy	Relaxation length ($\mu_o r$)					
		10		15		20	
Water	1	18.0182	(27.1)	29.2114	(50.4)	41.2529	(82.2)
Water	3	7.2477	(8.63)	10.2714	(12.8)	13.3000	(17.0)
Aluminum	1	16.7928	(21.2)	27.0853	(37.9)	38.1680	(58.5)
Concrete	1	16.0163		25.7455		36.4411	
Iron	1	12.7802	(16.2)	20.2085	(28.3)	28.1921	(42.7)

*Figures in the parentheses are those results reported in NYO 3075, AEC, June 30, 1954
"Calculations of the penetration of Gamma Rays"
Final Report
By
Herbert Goldstein
Ernest J. Wilkins, Jr.

TABLE 9

Linear Attenuation Coefficient

Total Dose ($D_o^o + D_o^s$)

Medium:	Energy - μ_o (cm^{-1})	$\mu_o r$	r (cm)	$4\pi r^2$	$e^{\mu_o r}$	$\left(4\pi r^2 \quad e^{\mu_o r}\right)$
Water	1.0	0.0706				
		1	14.1643	2521.4851	2.7183	$6.8542 \times 10^{+3}$
		2	28.3286	10085.9404	7.3891	$74.5257 \times 10^{+3}$
		4	56.6572	40343.7615	54.5982	$2202.6989 \times 10^{+3}$
		7	99.1501	123552.7696	1096.6332	135.4922×10^{6}
		10	141.6431	252148.8655	22026.4658	55.5396×10^{8}
		15	212.4646	567334.6802	3269017.372	18.5462×10^{11}
		20	283.2861	1008594.7502	485165195.4	4.8933×10^{14}

$\mu_o r$	Computer Output for Dose	Total Dose (Mev/cm^3 sec)	Total Dose (r/hr) (Mev/cm^3 sec)/19.6
1	$0.71726085 \times 10^{-4}$	1.0465×10^{-8}	0.0534×10^{-8}
2	$0.11797986 \times 10^{-3}$	0.15831×10^{-8}	0.0081×10^{-8}
4	$0.23034650 \times 10^{-3}$	1.0458×10^{-10}	0.0534×10^{-10}
7	$0.42822049 \times 10^{-3}$	3.1605×10^{-12}	0.1613×10^{-12}
10	$0.64955419 \times 10^{-3}$	1.1695×10^{-13}	0.0597×10^{-13}
15	$0.10530706 \times 10^{-2}$	0.5678×10^{-14}	0.0290×10^{-14}
20	$0.14871652 \times 10^{-2}$	3.0392×10^{-17}	0.1551×10^{-17}

TABLE 10

Energy (Mev)	Absorption Coeff. (Thompson unit/electron)	Absorption Coeff. (cm^2/gm)	Photon Wave Length (Compton Unit)
0.0100	132.1160	24.6000	51.1000
0.0200	17.2141	3.3400	25.5500
0.0300	5.5742	1.1000	17.0333
0.0400	2.7877	0.5420	12.7750
0.0500	1.8007	0.3500	10.2200
0.0600	1.3636	0.2670	8.5167
0.0800	1.0010	0.1970	6.3875
0.1000	0.8511	0.1690	5.1100
0.1500	0.6955	0.1390	3.4067
0.2000	0.6249	0.1240	2.5550
0.3000	0.5363	0.1070	1.7033
0.4000	0.4778	0.0954	1.2775
0.5000	0.4357	0.0870	1.0220
0.6000	0.4163	0.0804	0.8517
0.8000	0.3535	0.0706	0.6387
1.0000	0.3177	0.0635	0.5110
1.5000	0.2589	0.0517	0.3407
2.0000	0.2232	0.0445	0.2555
3.0000	0.1820	0.0363	0.1703
4.0000	0.1592	0.0317	0.1277
5.0000	0.1447	0.0290	0.1022
6.0000	0.1346	0.0270	0.0852
8.0000	0.1224	0.0245	0.0639
10.0000	0.1154	0.0231	0.0511

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APPENDIX I

MAIN

```
.PROGRAM FOR CALCULATING THE MOMENTS FOR PLAIN
.-------------------------------------------------
.ISOTROPIC SOURCE  WITH P-7 APPROXIMATION.MOMENTS FOR
.-------------------------------------------------
.POINT ISOTROPIC SOURCE HAS BEEN OBTAINED FROM
.-------------------------------------------------
.THE MOMENTS CALCULATED FOR PLANE ISOTROPIC SOURCE.
.-------------------------------------------------
.THESE MOMENTS HAVE BEEN USED TO CONSTRUCT THE GAMMA
.-------------------------------------------------
.FLUX,TOTAL DOSE AND DOSE BUILD UP FACTOR.
.   COHERENT SCATTERING HAS BEEN EXCLUDED.
.-------------------------------------------------
     ENRGY = PHOTON ENERGY
     LAMDA = DIMENSIONLESS PHOTON WAVE-LENGYH AT
.ENERGY,'ENRGY'.
     MU =  ABSORPTION COEFF. OF THE SCATTERING MEDIUM.
     ENRGY AND MU ARE USED ONLY TO INTERPOLATE THE
.DATA ON ABSORPTION COEFF.FOR THE MEDIUM.
     LMDAO = INCIDENT PHOTON WAVE LENGTH.
     MUO = ABSORPTION COEFF. OF THE MEDIUM AT THE
 INCIDENT GAMMA ENERGY.
     DEL = INCREMENT IN LAMDA VALUE, ' LMDA '.
     B0 =  MOMENTS AT LMDA = LMDAO.
     B1 =  MOMENTS AT LMDA = LMDAO +DEL.
*****INTERPP IS THE SUBROUTINE WHICH CALCULATES THE
 THE INTERPOLATED DATA ON MU.
     ANS =  INTERPOLATED ABSORPTIO COEFF. AT GAMMA
 ENERGY,'E' AND WAVE-LENGTH 'LMDA'.
****** HLMDA FUNCTION SUBPROGRAMME CALCULATES PRODUCT
 OF KLINE-NISHINA KERNAL AND LEGENDRE FUNCTION,P_L(X),
 X=1.+LMDAO-LMDA.
     KAUNT = ALL 36 NON- ZERO MOMENTS ARE COUNTED BY
 IT IN A LOGIC.
*****LOCATE = THIS FUNCTION SUB- PROGRAMME LOCATED
 36*NUMBER OF NON-ZERO MOMENTS ALONG THE MAIN DIAGONALS.
*****BNLO FUNCTION SUBPROGRAMME CALCULATES THE MOMENTS
 AT INCIDENT GAMMA ENERGY ,'EO'.
*****C(N,L) = THIS FUNCTION CALCULATES THE C(N,L) DATA
FOR THE PLAIN ISOTROPIC SOURCE.
*****FAC FUNCTION CALCULATES THE FACTORIAL VALUE FOR
 THE FUNCTION C(N,L)
******BNLL FUNCTION SUBPROGRAMME CALCULATES THE MOMENT
 AT GAMMA WAVE LENGTH,LMDA=LMDAO+DEL.
*****TLMDA    FUNCTION SUBPROGRAMME CALCULATES  THE
 INTERMEDIATE STEPS IN FINDING THE MOMENTS.
*****KLNSH    THIS FUNCTION SUB-PROGRAMME CALCULATES
THE KLINE-NISHINA KERNEL, WHERE A=λ0/λ ,B= λ0 -λ
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MAIN

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*****PLEGD    IS THE FUNCTION SUBPROGRAMME WHICH
CALCULATES THE LEGENDRE FUNCTION FOR THE VARIABLE
X=(1.+LMDAO-LMDA)AND L IS THE ORDER OF POLYNOMIAL.
*****INTG     THIS FUNCTION CALCULATES THE MOMENTS BY
PERFORMING THE INTEGRATION FOR THE PRODUCT OF H(λ',λ)
AND THE PRECEEDING MOMENTS,V(I).
*****INT      THIS SUBROUTINE IS CALLED BY THE FUNCTION
SUBPROGRAM INTG TO PERFORM THE ACTUAL INTEGRATION
 BY SIMPSON'S RULE AND TRAPIZOIDAL RULE COMBINED.
 NOTE THAT IN THE ARGUMENT LIST OF THE FUNCTION
SUBPROGRAM  INTG,I HAS BEEN USED. THIS I KEEPS TRACK
OF THE EVEN OR ODD NUMBER OF INTERVALS.
*****VALUE1    PERFORMS THE SAME JOB AS THE FUNCTION
SUBPROGRAM 'INTG'. THE DIFFERENCE BEING IT'S USED
OUTSIDE THE PRIMARY SCATTERING REGION, GREATER THAN
2.0+λo OR  GREATER THAN 4.0+λo .
*****VALUE2 AND *** VALUE3 FUNCTION SUBPROGRAMME DOES
THE SAME JOB AS THE  FUNCTION INTG. VALUE1 IS USED
TO ACCOUNT FOR THE SUBSCRIPTED VARIABLES HPP WHICH
ARE THE PRODUCT OF  KLINE-NISHINA KERNAL AND LEGENDRE
POLYNOMIAL,TIMES THE MOMENT.SO, TO START AT THE
BEGINNING,50 HPP VALUES ARE CALLED BY THE FUNCTION
SUBPROGRAMME VALUE1 BETWEEN THE LIMIT λ=LAMDA1 TO
λ=LAMDA1+2.
          THE OUTPUT OF VALUE1 GIVES THE MOMENT
AT OR OUTSIDE λ = λo+4.0
NOW, VALUE2 WOULD PERFORM THE INTREGRATION FOR THOSE
POINTS WHICH ARE NEEDED TO MAKE ALTOGATHER 50 POINTS.
PRECISELY, WHEN ALL THE MOMENTS ARE OBTAINED AT THE
PRIMARY SCATTERING REGION,WE STILL WOULD NEED,
FOR THIS CHOICE OF Δλ=.04, 50 POINTS.,EXCLUDING THE
VARY FIRST ONE.AT THE POINT λ = λo+2.0+.08, WE WOULD
NEED 50 POINTS. OUTSIDE λ =λo+2.0, WE HAVE 2 POINTS,
ONE AT λ = λo+2.+.04 & THE OTHER AT λ =λo+2.+.08
***VALUE2 WOULD TAKE CARE OF THESE POINTS AND VALUE1
WOULD TAKE CARE OF OF REST OF THE 49 POINTS(HPP).
-----VALUE3 FUNCTION SUBPROGRAM ESSENTIALLY THE SAME
AS THE ****VALUE2
---------- PROGRAM WRITE-UP. ----------
      DIMENSION LAMDA(50),ENRGY(50),MU(50),V(301),
     1HP(301),BNL(36,301),BNL2(8,301),BNL3(301),
     2AFENRG(25),ARMU(25),HPP(301),HPPP(301),DV(2),
     3HPPPP(301)
      REAL* 4 LAMDA,MU,LMDAO,MUO,KNK,LMDA,KLNSH,INTG,
     1MUOAIR,LAMDA1,LAMDA2,LAMDA3
      NN=48
      READ(5,10)(LAMDA(I),I=1,NN)
10    FORMAT(16F5.0/16F5.0/16F5.0)
```

```
      READ(5,20) (MU(J),J=1,NN)
20    FORMAT(13F6.0/13F6.0/13F6.0/9F6.0)
      DO 30 I=1,NN
      ENRGY(I)=0.511/LAMDA(I)
30    CONTINUE
      DO 36 K = 1,25
      READ(5,35) ARENRG(K),ARMU(K)
35    FORMAT(F10.0,E12.4)
36    CONTINUE
C     LAMDA1 IS THE WAVE-LENGTH AT THE START. IT IS
CEQUAL TO LAMDA0.
C     LAMDA2=LAMDA1+2. AND LAMDA3=LAMDA1+4.
C     LIMIT1 IS THE REGION WHERE PRIMARY SCATTERING  IS
COVER.
C     LIMIT1 IS THE REGION WHERE LAMDA2 STARTS.
C     LIMIT2 IS THE REGION WHERE LAMDA3 STARTS.
C     LIMIT IS THE NUMBER - 1, THAT IS, TOTAL NUMBER OF
CPOINTS MINUS 1.
C*****PRIMARY SCATTERING REGION  ----
C******SECONDARY SCATTERING REGION-----
C******SECONDARY SCATTERING REGION-----
9999  READ(5,40) LMDA0,MU0,RMU,DEL,LIMIT1,LIMIT2,LIMIT,
     1EC,MU0AIR,LAMDA1,LAMDA2,LAMDA3
40    FORMAT(4F10.0,3I5,F10.0,E12.4/3F10.0)
      IF(LMDA0.EQ.0.0) GO TO 7003
C CALCULATING THE MOMENT FOR PLAIN ISOTROPIC SOURCE.
      NK=1
      LK=1
      KAUNT = 1
      GO TO 56
55    NK = NK + 1
      LK = LK + 1
57    N = NK - 1
      L = LK - 1
      Y=C(N,L)
      E = E0
      LMDA0=LAMDA1
      B0 = BNL0(Y,LMDA0,MU0,N,L)
C----- CALCULATING THE MOMENT AT LAMDA(0)-------------
      I=1
      BNL( LOCATE(NK,LK),1) = B0
      V(1)=B0
      IF(KAUNT.EQ.1) BNL2(1,1) = B0
C     KAUNT IS THE VARIABLE WHICH KEEPS TRACK OF THE
C     MOMENTS  ALONG THE DIAGONAL, FOR EXAMPLE,
C     KAUNT EQ. 1 TO 8 ...FIRST DIAGONAL
C     KAUNT EQ. 9 TO 15..SECOND DIAGONAL
C     .............................
```

```
C       KAUNT EQ.34 TO 35.SEVENTH DIAGONAL
C       KAUNT EQ.36          EIGHTH DIAGONAL
        IF(KAUNT.EQ.9) BNL2(2,1) = BO
        IF(KAUNT.EQ.16)BNL2(3,1) = BO
        IF(KAUNT.EQ.22)BNL2(4,1) = BO
        IF(KAUNT.EQ.27)BNL2(5,1) = BO
        IF(KAUNT.EQ.31)BNL2(6,1) = BO
        IF(KAUNT.EQ.34)BNL2(7,1) = BO
        IF(KAUNT.EQ.36)BNL2(8,1) = BO
C-----  CALCULATING THE MOMENT AT LAMDA(0)+DEL
        I=2
        LMDA = LMDAO + DEL
        E=.511/LMDA
        A=LMDAO/LMDA
        B=LMDAO-LMDA
        X = 1.+LMDAO-LMDA
        KNK = KLNSH(A,B)
        P=PLEGD(X,L)
        H = HLMDA(KNK,P)
        Y=C(N,L)
        T = TLMDA(LMDAO,MUO,H,Y,N,L,KAUNT,BNL,I)
C-----THE ARGUMENT LIST IN THE SUBROUTINE INTERP IS
CSUBJECT TO CHANGE.FOR EXAMPLE,THE LAST NUMBER IN THE
CARGUMENT LIST,SUCH AS,24 OR 35 OR 48 REPRESENTS THE
CNUMBER OF DATA POINTS FOR  LAMDA(I) OR MU(J).
        CALL INTERP(E,ENRGY,MU,ANS,3,48)
        B1 = BNL1(T,H,ANS,DEL,BO)
        BNL( LOCATE(NK,LK),2) = B1
        V(2)=B1
        IF(KAUNT.EQ. 1)BNL2(1,2) = B1
        IF(KAUNT.EQ. 9)BNL2(2,2) = B1
        IF(KAUNT.EQ.16)BNL2(3,2) = B1
        IF(KAUNT.EQ.22)BNL2(4,2) = B1
        IF(KAUNT.EQ.27)BNL2(5,2) = B1
        IF(KAUNT.EQ.31)BNL2(6,2) = B1
        IF(KAUNT.EQ.34)BNL2(7,2) = B1
        IF(KAUNT.EQ.36)BNL2(8,2) = B1
C-----WE START AT  LMDA=0.591 AND STOP AT LMDA=6.511--
        NUMBER=LIMIT+1
        DO 500 I = 3,NUMBER
        LAMDAO=LAMDA1
        J = I - 2
        LMDA = LMDA + DEL
        E=.511/LMDA
        CALLINTERP (E,ENRGY,MU,ANS,3,48)
        IF(I.GT.LIMIT1.AND.I.LE.LIMIT2) GO TO 496
        IF(I.EQ.LIMIT2+1) GO TO 150
        IF(I.GT.LIMIT2+1) GO TO 159
```

```
      X = 1.+LMDA0-LMDA
      A=LMDA0/LMDA
      B=LMDA0-LMDA
      Y=C(N,L)
      KNK = KLNSH(A,B)
      P=PLEGD(X,L)
      H = HLMDA(KNK,P)
      T = TLMDA(LMDA0,MU0,H,Y,N,L,KAUNT,BNL,I)
      V(I) = (T + INTG(I,J,B0,B1,H,V,LMDA0,LMDA,
     1KNK,P,X,L,DEL,LAMDA1,LIMIT1))/(ANS-DEL/4.)
      IF(I.LT.LIMIT1) GO TO 490
      IF(I.EQ.LIMIT1) GO TO 491
C-----THIS STEP CALCULATES ALL THE 50 HP VALUES
C-----STARTINGLMDA=.551 UPTO 2.511 THESE HP-VALUES ARE
C-----USED TO CALCULATE MOMENTS BETWEEN LAMDA2 TO LAMDA3
491   M2 = 1
      M3 = LIMIT1-1
      LMDA0 = LMDA0+DEL
      X = 1. + LMDA0 - LMDA
      A = LMDA0/LMDA
      B=LMDA0-LMDA
      KNK = KLNSH(A,B)
      P = PLEGD(X,L)
      H = HLMDA(KNK,P)
      MM=2
493   DO 495 M1 = M2,M3
      IF(M1.GT.1) GO TO 494
      HP(M1)=H*V(MM)
      GO TO 495
494   LMDA0 = LMDA0+DEL
      X = 1. + LMDA0 - LMDA
      A = LMDA0/LMDA
      B=LMDA0-LMDA
      KNK = KLNSH(A,B)
      P = PLEGD(X,L)
      H = HLMDA(KNK,P)
      MM=MM+1
      HP(M1)=H*V(MM)
      IF(M1.EQ.M3) LMDA0=LAMDA1
495   CONTINUE
      IF(I.EQ.LIMIT1) GO TO 141
496   IF(I.EQ.LIMIT2) GO TO 499
      IF(I.EQ.LIMIT1+1) GO TO 142
      IF(I.EQ.LIMIT2+1) GO TO 142
      IF(I.GT.LIMIT1+1.AND.I.LT.LIMIT2) GO TO 142
141   LIMIT4=1
      GO TO 143
142   LIMIT4=LIMIT4+1
```

```
      IF(I.GT.LIMIT1)   GO TO 144
143   IF(I.EQ.LIMIT1) LIMIT3=LIMIT1-1
      GO TO 145
144   LIMIT3=LIMIT3-1
      IF(I.EQ.LIMIT2) GO TO 499
145   M5 = LIMIT1-1
C-----THESE HP,S WERE CALCULATED BETWEEN STATEMENT NO.
C491 AND 495 .TO INTEGRATE THESE HP'S WE ARE
CTRANSFORMING THEM TO A NEW VARIABLE HPP AND USED THE
C-----FUNCTION SUBPROGRAMME  **** VALUE1 *******
      KD=1
      DO 146 M4=LIMIT4,M5
      DUMMY=HP(M4)
      HPP(KD)=DUMMY
      KD=KD+1
146   CONTINUE
      IF(I.EQ.LIMIT1) GO TO 497
      IF(I.EQ.LIMIT1+1) GO TO 498
      IF(I.GT.LIMIT1+1.AND.I.LE.LIMIT2) GO TO 499
C-----CALCULATING THE MOMENT AT THE FIRST DISCONTINUOUS
CPOINT,WHERE,    +2.0. THIS MOMENT IS IDENTIFIED AS DV(1)
497   DV(1)=VALUE1(LIMIT3,HPP,DEL)/(ANS-DEL/4.)
      GO TO 490
C-----CALCULATING THE MOMENT AT LAMDA(0)+2.0+DEL. HERE
CWE USED 49 HPP VALUES,SINCE WE NEED TO INTEGRATE
CALWAYS 51 POINTS, FOR THIS CHOICE OF DEL(.04),WE NEED
CHP AT    TO HP AT    FOR INTEGRATION.NOTE THAT,SINCE
CMOMENT V(51) AND DV(1) FALL ON THE SAME POINT,THE
CINTEGRATION OF  H*V(51) AND H*DV(1) IS ZERO.SO 49
CPOINTS HAVE BEEN USED.
498   H=0.
      T = TLMDA(LMDA0,MU0,H,Y,N,L,KAUNT,BNL,I)
      V(I)=(T+VALUE1(LIMIT3,HPP,DEL))/(ANS-DEL/4.)
      GO TO 490
CBETWEEN THE STATEMENT NO.499 AND 711 WE ARE GENERATING
CTHE PRODUCT OF KLINE-NISHINA KERNEL(KNK),LEGENDRE
CPOLYNOMIAL & THE CORRESPONDING MOMENT,V(I).THE PRODUCT
CIS STORED AS HPPP(KD1) AND ARE USED BY THE FUNCTION
CSUBPROGRAM ** VALUE2 ***
499   LMDA0=LAMDA2
      KD2=I-LIMIT1
      J=KD2-1
      DO701 KD1=1,KD2
      IF(KD1.EQ.2) GO TO 702
      IF(KD1.GT.2) GO TO 705
      GO TO 709
702   KD3=LIMIT1+1
      GO TO 710
```

```
705     KD3=KD3+1
        GO TO 710
710     IF(KD1.GT.1) LMDAO=LMDAO+DEL
709     X = 1. + LMDAO - LMDA
        A = LMDAO/LMDA
        B=LMDAO-LMDA
        KNK = KLNSH(A,B)
        P = PLEGD(X,L)
        H = HLMDA(KNK,P)
        IF(KD1.EQ.1) GO TO 700
        GO TO 711
700     HPPP(KD1)=H*DV(1)
        GO TO 701
711     HPPP(KD1)=H*V(KD3)
        IF(KD1.EQ.KD2) LMDAO=LAMDA2
701     CONTINUE
        H=0.
        T = TLMDA(LMDAO,MUO,H,Y,N,L,KAUNT,BNL,I)
        IF(I.EQ.LIMIT2) GO TO 147
        VAL=VALUE1(LIMIT3,HPP,DEL)
        GO TO 148
C-----CALCULATING THE MOMENT BETWEEN LAMDA2+2*DEL AND
C-----LAMDA3
147     VAL=0.0
148     V(I)=(T+VAL+VALUE2(J,HPPP,DEL))/(ANS-DEL/4.)
        IF(I.EQ.LIMIT2) GO TO 149
        GO TO 490
149     LIMIT4=1
        GO TO 151
150     LIMIT4=LIMIT4+1
        GO TO 152
159     LIMIT4=LIMIT4+1
        LIMIT3=LIMIT3-1
        IF(I.EQ.NUMBER) GO TO 161
        GO TO 489
151     IF(I.EQ.LIMIT2) LIMIT3=LIMIT1-1
        GO TO 153
152     LIMIT3=LIMIT3-1
        GO TO 155
153     M8=1
        M9=LIMIT2-LIMIT1
        LMDAO=LAMDA2+DEL
        X = 1. + LMDAO - LMDA
        A = LMDAO/LMDA
        B=LMDAO-LMDA
        KNK = KLNSH(A,B)
        P = PLEGD(X,L)
        H = HLMDA(KNK,P)
```

```
      MM=LIMIT1+1
      DO 716 M7=M8,M9
      IF(M7.GT.1) GO TO 714
      HPPP(M7)=H*V(MM)
      GO TO 716
714   LMDAO=LMDAO+DEL
      X = 1. + LMDAO - LMDA
      A = LMDAO/LMDA
      B=LMDAO-LMDA
      KNK = KLNSH(A,B)
      P = PLEGD(X,L)
      H = HLMDA(KNK,P
      MM=MM+1
      HPPP(M7)=H*V(MM)
      IF(M7.EQ.M9) LMDAO=2.511
716   CONTINUE
C-----THIS STEP REDEFINES THE VARIABLES HPPP(KD) WHICH
C-----WERE GENERATED BETWEEN  THE STATEMENT NO. 153
C-----716 AT I=101 AND SETS EQ.HPP. THIS 50 HPP VALUES
C-----ARE USED TO CALCULATE MOMENTS BETWEEN  =4.511
C-----AND 6.511.
      DO 154 KD=1,50
      HPP(KD)=HPPP(KD)
154   CONTINUE
C-----CALCULATING THE MOMENT AT THE SECOND DISCONTINU-
C-----OUS POINT. THIS CORRESPOND TO  =4.511
487   DV(2) = VALUE1(LIMIT3,HPP,DEL)/(ANS-DEL/4.)
      GO TO 490
155   M5=LIMIT1-1
      K1=1
      DO 157 M4=LIMIT4,M5
      DUMMY1=HPP(M4)
      HPP(K1)=DUMMY1
      K1=K1+1
157   CONTINUE
C     CALCULATING THE MOMENT AT THE POINT LIMIT2+1
488   H=0.
      T = TLMDA(LMDAO,MUO,H,Y,N,L,KAUNT,BNL,I)
      V(I)=(T+VALUE1(LIMIT3,HPP,DEL))/(ANS-DEL/4.)
      GO TO 490
489   M5=LIMIT1-1
      K2=1
      DO 160 M4=LIMIT4,M5
      DUMMY2=HPP(M4)
      HPP(K2)=DUMMY2
      K2=K2+1
160   CONTINUE
161   LMDAO=LAMDA3
```

MAIN

```
      KD8=I-LIMIT2
      J=KD8-1
      DO 704 KD7=1,KD8
      IF(KD7.EQ.2) KD9=LIMIT2+1
      IF(KD7.GT.2) KD9=KD9+1
      IF(KD7.GT.1)  LMDAO = LMDAO+DEL
      X = 1. + LMDAO - LMDA
      A=LMDAO/LMDA
      B=LMDAO-LMDA
      KNK=KLNSH(A,B)
      P=PLEGD(X,L)
      H=HLMDA(KNK,P)
      IF(KD7.EQ.1) GO TO 712
      HPPPP(KD7) = H * V(KD9)
      IF(KD7.EQ.KD8) LMDAO=LAMDA3
      GO TO 704
712   HPPPP(KD7)=H*DV(2)
704   CONTINUE
      H=0.
      T = TLMDA(LMDAO,MUO,H,Y,N,L,KAUNT,BNL,I)
      IF(I.EQ.NUMBER) GO TO 162
      VAL=VALUE1(LIMIT3,HPP,DEL)
      GO TO 163
162   VAL=0.0
C     CALCULATING THE MOMENT BETWEEN LIMIT2+2 AND NUMBER
C****** VALUE3 ***** FUNCTION SUBPROGRAMME IS USED TO
C     INTREGATE THE POINTS HPPPP(KD7). HPPPP(KD7) ARE
C     REQUIRED FOR INTREGATION BETWEEN LIMIT2 AND NUMBER
163   V(I)=(T+VAL+VALUE3(J,HPPPP,DEL))/(ANS-DEL/4.0)
490   BNL(LOCATE(NK,LK),I) = V(I)
      IF(KAUNT.EQ.1) BNL2(1,I) = V(I)
      IF(KAUNT.EQ.9) BNL2(2,I) = V(I)
      IF(KAUNT.EQ.16)BNL2(3,I) = V(I)
      IF(KAUNT.EQ.22)BNL2(4,I) = V(I)
      IF(KAUNT.EQ.27)BNL2(5,I) = V(I)
      IF(KAUNT.EQ.31)BNL2(6,I) = V(I)
      IF(KAUNT.EQ.34)BNL2(7,I) = V(I)
      IF(KAUNT.EQ.36)BNL2(8,I) = V(I)
500   CONTINUE
      LAMDAO=LAMDA1
      LMDA = LMDAO + DEL
      KAUNT = KAUNT + 1
      IF(KAUNT.LE.8) GO TO 55
      IF(KAUNT.GT.8.AND.KAUNT.LE.15) GO TO 501
      GO TO 504
501   IF(KAUNT.EQ.9) GO TO 502
      GO TO 503
502   NK = 3
```

MAIN

```
      LK = 1
      LAMDAO=LAMDA1
      GO TO 56
503   IF(KAUNT.LE.15) GO TO 55
504   IF(KAUNT.GT.15.AND.KAUNT.LE.21) GO TO 505
      GO TO 508
505   IF(KAUNT.EQ.16) GO TO 506
      GO TO 507
506   NK = 5
      LK = 1
      LAMDAO=LAMDA1
      GO TO 56
507   IF(KAUNT.LE.21) GO TO 55
508   IF(KAUNT.GT.21.AND.KAUNT.LE.26) GO TO 509
      GO TO 512
509   IF(KAUNT.EQ.22) GO TO 510
      GO TO 511
510   NK = 7
      LK = 1
      LAMDAO=LAMDA1
      GO TO 56
511   IF(KAUNT.LE.26) GO TO 55
512   IF(KAUNT.GT.26.AND.KAUNT.LE.30) GO TO 513
      GO TO 516
513   IF(KAUNT.EQ.27) GO TO 514
      GO TO 515
514   NK = 9
      LK = 1
      LAMDAO=LAMDA1
      GO TO 56
515   IF(KAUNT.LE.30) GO TO 55
516   IF(KAUNT.GT.30.AND.KAUNT.LE.33) GO TO 517
      GO TO 520
517   IF(KAUNT.EQ.31) GO TO 518
      GO TO 519
518   NK = 11
      LK = 1
      LAMDAO=LAMDA1
      GO TO 56
519   IF(KAUNT.LE.33) GO TO 55
520   IF(KAUNT.GT.33.AND.KAUNT.LE.35) GO TO 521
      GO TO 524
521   IF(KAUNT.EQ.34) GO TO 522
      GO TO 523
522   NK = 13
      LK = 1
      LAMDAO=LAMDA1
      GO TO 56
```

MAIN

```
523     IF(KAUNT.LE.35) GO TO 55
524     IF(KAUNT.EQ.36) GO TO 525
        IF(KAUNT.GT.36) GO TO 528
525     NK = 15
        LK = 1
        LAMDAO=LAMDA1
        GO TO 56
528     LAMDAO=LAMDA1
934     FORMAT(1H1,////)
        WRITE(6,934)
        WRITE(6,905) RMU
905     FORMAT(1X,7X,'DEPTH OF PENETRATION ',F3.0/)
C-----TO RECONSTRUCT THE SCATTERED ENERGY FLUX, WE NEED
C       MOMENTS,STARTING N=0 TO N=7,,THAT IS,B(0,0),
C       B(2,0),B(4,0),.........B(14,0). ALL THESE MOMENTS
C-----ARE REQUIRED AT EACH ENERGY  OR AT EACH PHOTON
C-----WAVE LENGTH BETWEEN  =0.511 AND 6.511.
C-----I=151=NUMBER,COVERS FROM  =0.511 TO  =6.511,DEL=.04
        WRITE(6,930)
930     FORMAT(9X,'ENERGY ( MEV ) ',13X,'DIFFERENTIAL'/
       137X,'ANGULAR ENERGY'/37X,'FLUX'/)
        LMDA=LAMDA1
        DO 940 I = 1,NUMBER
        Q = 0.
        KM=1
C-----CALCULATING THE BIORTHONORMAL POLYNOMIAL
        DO 920 J =1,8
        N1=J-1
        QN2N = 0.
        DO 910 K =KM,8
        N2=K-1
        PLG = PLEGUR(RMU,N2)
        CAL = FAC(N2)/(FAC(N1) * FAC(N2 - N1))
        QN2N = QN2N + CAL * PLG * (-1.)**N1
910     CONTINUE
        KM=KM+1
        Q = QN2N * ( 2. * N1 + 1. ) * BNL2 ( J,I) + Q
920     CONTINUE
        E=0.511/LMDA
        CALL INTERP ( E,ARENRG,ARMU,ARMUI,3,25)
        BNL3(I)= (Q*ARMUI*.511)/(LMDA**2)
        LMDA=LMDA+DEL
        WRITE(6,932) E,Q
932     FORMAT(13X,F7.4,11X,E20.8)
        PUNCH 933, E,Q
933     FORMAT(F7.4,3X,E20.8)
940     CONTINUE
C-----CALCULATING THE DOSE AND DOSE BUILD UP FACTOR----
```

MAIN

```
C-----NOTE THAT DOSE HAS THE UNIT OF MEV/SQ.CM.SEC. CM
      BNL4=MUOAIR*.511/LAMDA1
      CALL SIMPS(ANS2,BNL3,DEL,LIMIT)
      BNL5=ANS2
      BILDUP=1. +(BNL5/BNL4)
      DOSE= BNL4*BILDUP
      WRITE (6,1160) DOSE, BILDUP
1160  FORMAT(15X,'DOSE ',E20.8,11X,'BUILD UP FACTOR '
     1,E20.8//)
      GO TO 9999
7003  STOP
      END
```

INTERP

```
      SUBROUTINE INTERP(XIN,X,F,ANS,NPTS,IMAX)
      DIMENSION X(IMAX),F(IMAX),XN(12),FN(12)
      XUP = 1.E30
      DO 11 I=1,IMAX
      T = XIN - X(I)
      IF (T) 8,9,9
    8 T =-T
    9 IF (T-XUP) 10,11,11
   10 IP = I
      XUP = T
   11 CONTINUE
      IN = 1
      NDEG = NPTS - 1
      DO 18 I = 1,NPTS
      FN(I) = F(IP)
      XN(I) = X(IP)
      IF (IN) 12,12,13
   12 IQ = IP - I
      GO TO 15
   13 IQ = IP + I
      IF (IMAX - IQ) 14,15,15
   14 IP =IP - 1
      GO TO 18
   15 IF (IQ) 16,16,17
   16 IP = IP + 1
      GO TO 18
   17 IP = IQ
      IN = -IN
   18 CONTINUE
      ANS = 0.
      FACT = 1.
      DO 20 J = 1,NDEG
      ANS = ANS + FACT*FN(1)
      DO 19 I = J,NDEG
      IQ = I - J + 1
   19 FN(IQ) = (FN(IQ + 1) - FN(IQ))/(XN(I+1) - XN(IQ))
   20 FACT = FACT*(XIN-XN(J))
      RETURN
      END
```

CAC

```
      FUNCTION C(N,L)
      C=0.
      M=N-L
      IF(N.LT.L) RETURN
      IF(M-M/2*2.NE.0)RETURN
      IF(N-N/2*2.EQ.1.AND.L.EQ.0) RETURN
      IF(N-N/2*2.EQ.0.AND.L.EQ.0) GO TO 30
      C= 2.**L*FAC(N)*FAC((N+L)/2)/FAC((N-L)/2)/FAC((N+L+1))
      RETURN
30    C=1./(N+1.)
      RETURN
      END
```

FAC

```
      FUNCTION FAC(N)
      FAC=1.0
      IF (N.LE.1) RETURN
      DO 50 I=2,N
50    FAC = FAC*I
      RETURN
      END
```

BNL1

```
 FUNCTION BNL1 (T,H,ANS,DEL,BO)
BNL1=(T+H*BO*DEL/2.)/(ANS-3.*DEL/8.)
RETURN
END
```

TLMDA

```
      FUNCTION TLMDA (LMDAO,MUO,H,Y,N,L,KAUNT,BNL,I)
      REAL * 4 LMDAO,MUO
      DIMENSION BNL(36,301)
      PL = MUO/(2.*L+1.)
      QL = PL*L
      RL = PL*(L + 1.)
      NN = N + 1
      KK = L + 1
      Z1 = LMDAO*H*Y
      IF(I.GT.51) Z1=0.
      TLMDA = Z1
      IF(NN.EQ.1) RETURN
      IF(NN.NE.KK) GO TO 999
      TLMDA = Z1 +BNL(LOCATE(NN-1,KK-1),I)*QL
      IF(NN.EQ.KK) RETURN
999   JJ = NN - KK
      IF(JJ.EQ.2.OR.JJ.EQ.4.OR.JJ.EQ.6.OR.JJ.EQ.8.OR.JJ
     1.EQ.10.OR.JJ.EQ.12.OR.JJ.EQ.14) GO TO 1000
1000  IF(KAUNT.EQ.9.OR.KAUNT.EQ.16.OR.KAUNT.EQ.22.OR.
     1KAUNT.EQ.27.OR.KAUNT.EQ.31.OR.KAUNT.EQ.34.OR.
     2KAUNT.EQ.36) GO TO 1001
      GO TO 1002
1001  TLMDA = Z1 + RL*BNL(LOCATE(NN-1,KK+1),I)
      RETURN
1002  TLMDA = Z1 + RL*BNL(LOCATE(NN-1,KK+1),I) +QL*BNL(
     1LOCATE(NN-1,KK-1),I)
      RETURN
      END
```

LOCATE DATE = 71064

```
      FUNCTION LOCATE(NK,LK)
      IA = -9
      DO 7000 LLL=1,NK,2
      IA = IA + (9 - LLL/2)
7000  CONTINUE
      LOCATE = IA + LK
      RETURN
      END
```

BNLO

```
      FUNCTION BNLO ( Y,LMDAO,MUO,N,L)
      REAL*4 LMDAO,MUO
      BNLO = 0.75*LMDAO/MUO
      IF(N-N/2*2.EQ.0.AND.L.EQ.0) RETURN
      BNLO = 0.75*(N+1.)*LMDAO/MUO*Y
      RETURN
      END
```

INTG

```
      REAL FUNCTION INTG( I,J,B0,B1,H,V,LMDA0,LMDA,
     1KNK,P,X,L,DEL,LAMDA1,LIMIT1)
      REAL*4 KNK,LMDA0,LMDA,KLNSH,LAMDA1
      DIMENSION V(301),HP(301)
      J1 = J + 1
      DO 600 M=1,J1
      IF(M.GT.1) GO TO 597
      HP(M)=H*V(M)
      GO TO 600
597   LMDA0 = LMDA0 + DEL
      X = 1.+LMDA0-LMDA
      A=LMDA0/LMDA
      B=LMDA0-LMDA
      KNK = KLNSH(A,B)
      P=PLEGD(X,L)
      H = HLMDA(KNK,P)
      HP(M)=H*V(M)
      IF(I.LE.LIMIT1) GO TO 598
      GO TO 600
598   IF(M.EQ.J1) LMDA0=LAMDA1
600   CONTINUE
      IF(I-I/2*2.NE.0)GO TO 10
      GO TO 450
10    CALL INT   (ANS1,HP,DEL,J)
      INTG = ANS1
      RETURN
450   CALL SIMPS (ANS2,HP,DEL,J)
      INTG = ANS2
      RETURN
      END
```

HLMDA

```
      FUNCTION HLMDA (KNK,P)
      REAL * 4 KLNSH,KNK
      HLMDA = KNK*P
      RETURN
      END
```

KLNSH

```
      REAL FUNCTION KLNSH(A,B)
      KLNSH = (3./8.)*A*((1./A)+A+2.*B+B**2)
      RETURN
      END
```

PLEGD

```
      FUNCTION PLEGD(X,L)
      X2=X*X
      IF(L.EQ.0) GO TO 60
      GO TO (70,80,90,100,110,120,130),L
60    PLEGD=1.0
      RETURN
70    PLEGD=X
      RETURN
80    PLEGD=(3.*X2 - 1.)/2.
      RETURN
90    PLEGD=X*(5.*X2-3.)/2.
      RETURN
100   PLEGD=(X2*(35.*X2-30.)+3.)/8.
      RETURN
110   PLEGD=
     1X*(15.+X2*(-70.+63.*(X2)))/8.
      RETURN
120   PLEGD=
     1(-5.+X2*(105.+X2*(-315.+X2*231.)))/16.
      RETURN
130   PLEGD=
     1X*(-35.+X2*(315.+X2*(-693.+429.*X2)))/16.
      RETURN
      END
```

VALUE1

```
      FUNCTION VALUE1(LIMIT3,HPP,DEL)
      DIMENSION HPP(301)
      J=LIMIT3-1
      IF(J.EQ.0) GO TO 607
      IF(J-J/2*2.EQ.0) GO TO 605
      GO TO 606
605   CALL SIMPS(ANS2,HPP,DEL,J)
      VALUE1 = ANS2
      RETURN
606   CALL INT (ANS1,HPP,DEL,J)
      VALUE1=ANS1
      RETURN
607   VALUE1 = 0.
      RETURN
      END
```

VALUE2

```
      FUNCTION VALUE2 (J,HPPP,DEL)
      DIMENSION HPPP(301)
      IF(J-J/2*2.EQ.0) GO TO 703
      CALL INT(ANS1,HPPP,DEL,J)
      VALUE2 = ANS1
      RETURN
703   CALL SIMPS (ANS2,HPPP,DEL,J)
      VALUE2 = ANS2
      RETURN
      END
```

VALUE3

```
      FUNCTION VALUE3 (J,HPPPP,DEL)
      DIMENSION HPPPP(301)
      IF(J-J/2*2.EQ.0) GO TO 801
      CALL INT (ANS1,HPPPP,DEL,J)
      VALUE3=ANS1
      RETURN
801   CALL SIMPS (ANS2,HPPPP,DEL,J)
      VALUE2=ANS2
      RETURN
      END
```

INTGUR

```
      SUBROUTINE INT(ANS1,Y,DELX,N)
      DIMENSION Y(301)
      I=1
      TRP=(DELX/2.)*(Y(I)+Y(I+1))
      N=N-1
      IF(N.EQ.0) GO TO 65
      IF(N.EQ.2) GO TO 86
      I=3
      J=N
      GO TO 6
      ENTRY SIMPS (ANS2,Y,DELX,N)
      IF(N.EQ.2) GO TO 85
      I=2
      J=N+1
6     SUM4=0.0
      SUM2=0.0
      IK=1
7     SUM4=SUM4+Y(I)
      SUM2=SUM2+Y(I+1)
      I=I+1
      IF(IK.EQ.N-3) GO TO 60
      IK=IK+2
      I=I+1
      GO TO 7
60    IF(J-J/2*2.EQ.0)GO TO 70
      GO TO 80
65    ANS1 = TRP
      RETURN
70    ANS1=(DELX/3.)*(4.*SUM4+2.*SUM2+Y(2)+Y(N+2)+4.*Y(
     1N+1))+TRP
      RETURN
80    ANS2=(DELX/3.)*(4.*SUM4+2.*SUM2+Y(1)+Y(N+1)+4.*Y(
     1N))
      RETURN
85    ANS2 = (DELX/3.)*(Y(1)+4.*Y(2)+Y(3))
      RETURN
86    ANS1 = (DELX/3.)*(Y(2)+4.*Y(3)+Y(4))+TRP
      RETURN
      END
```

PLEGUR

```
      FUNCTION PLEGUR (RMU,N2)
      Y1 = RMU
      IF(N2.EQ.0) GO TO 701
      GO TO (702,703,704,705,706,707,708),N2
701   PLEGUR = 1.0
      RETURN
702   PLEGUR = 0.5* (1.-Y1)
      RETURN
703   Y2 = Y1*Y1
      PLEGUR = (3.-5.*Y1+Y2)/8.
      RETURN
704   Y2 = Y1*Y1
      Y3 = Y2 * Y1
      PLEGUR = (15.-33.*Y1+12.*Y2-Y3)/48.
      RETURN
705   Y2 = Y1*Y1
      Y3 = Y2*Y1
      Y4 = Y3*Y1
      PLEGUR=(105.-279.*Y1+141.*Y2-22.*Y3+Y4)/384.
      RETURN
706   PLEGUR = (945.-Y1*(2895.-Y1*(1830.-Y1*(405.-
     1Y1*(35.-Y1)))))/3840.
      RETURN
707   PLEGUR = (10395.-Y1*(35685.-Y1*(26685.-Y1*(75
     100.-Y1*(930.-Y1*(51.-Y1))))))/46080.
      RETURN
708   PLEGUR = (135135.-Y1*(509985.-Y1*(435960.-Y1*
     1(146685.-Y1*(23310.-Y1*(1848.-Y1*(70.-Y1)))))))/
     1645120.
      RETURN
      END
```

```
      DIMENSION TX(8),TY(8),A(9),B(9),X(353),Y(353),SYM(4)
      DIMENSION BUF(4000)
      REAL*8 LABEL(2),TX,TY
      INTEGER SYM,PTYPE
      DATA BASE/4H    /
      DATA XORF/'*'/
      CALL PLOTS(BUF,4000)
      CALL PLOT (0.0,-5.0,-3)
      CALL PLOT (0.0,2.5,-3)
C     READ PARAMETER CARDS
C     FIRST CARD CONTAINS NG, THE NUMBER OF GRAPHS TO BE DRAWN.
C     THERE MUST BE NG SETS OF PARAM. CARDS AND W/DATA.
C     SECOND CARD. NP = NO. OF PLOTS FOR FIRST GRAPH.
C     THERE MUST BE NP SETS OF DATA FOR FIRST GRAPH.
C     XU = NO. OF UNITS /INCH ON X-AXIS.
C     XZERO = X-VALUE AT THE ORIGIN.
C     XEND = LARGEST VALUE OF THE ABSCISSA.
C     YZERO = Y-VALUE AT THE ORIGIN.
C     YEND = LARGEST VALUE OF THE ORDINATE.
C     XLEN = LENGTH, IN INCHES, OF THE ABSCISSA.
C     YLEN = LENGTH,IN INCHES,OF THE ORDINATES.
C     DELTA = FRACTION OF AN INCH DRAWN/UNIT ON ORDINATE.
C     TYPE OF PLOT
C     PTYPE =1 FOR CARTESIAN PLOT
C           =2 FOR SEMI-LOG PLOT, ORDINATE IS LOG AXIS
C     LABELX=X-COORDINATE VALUE WHERE LABEL BEGINS
C     LABELY=Y-COORDINATE VALUE WHERE LABEL BEGINS
C     IF LABELX & LABELY NOT GIVEN, LABEL BEGINS 2
C     INCHES FROM END OF PLOT & 2 INCHES FROM THE TOP.
C     IF PTYPE =1 NO VALUE IS NEEDED FOR DELTA
C     IPLOT=1,2,3    POINT PLOT.
C          =4,5,6    LINE PLOT
C          =7,8,9    POINT PLOT EVERY J-TH POINT
C     SYM(1) = SYMBOL USED FOR FIRST PLOT,IF ANY.
C     SYM(2) = SYMBOL USED FOR 2ND PLOT,IF ANY.
C     SYM(3) = SYMBOL USED FOR 3RD PLOT,IF ANY.
C     SYM(4) = SYMBOL USED FOR 4TH PLOT,IF ANY.
C     THIRD CARD  TX  ARRAY CONTAINING X-AXIS LABEL
C                 TY  ARRAY CONTAINING Y-AXIS LABEL
      READ(5,1)NG
    1 FORMAT(I1)
      DO 100 K=1,NG
      READ(5,2)NP,XU,XZERO,XEND,YZERO,YEND,XLEN,YLEN,DELTA,
     1PTYPE,IPLOT, SYM,TX,TY
    2 FORMAT(I1,I3,3F5.0,F10.2,3F5.0,F10.2/4A4/4A4)
      KOUNT=KOUNT+1
      DO 3 IPLTS=1,NP
```

APPENDIX II

```
      DIMENSION TX(8),TY(8),A(9),B(9),X(353),Y(353),SYM(4)
      DIMENSION BUF(6000)
      REAL*8 LABEL(2),TX,TY
      INTEGER SYM,PTYPE
      DATA FAKE/4H    /
      DATA XORE/'*'/
      CALL PLOTS(BUF,6000)
      CALL PLOT(0.0,-5.0,-3)
      CALL PLOT (0.0,2.0,-3)
C     READ PARAMETER CARDS
C     FIRST CARD CONTAINS NG, THE NUMBER OF GRAPHS TO BE DRAWN.
C     THERE MUST BE NG SETS OF PARAM. CARDS 2&3 W/DATA.
C     SECOND CARD. NP = NO. OF PLOTS FOR FIRST GRAPH.
C     THERE MUST BE NP SETS OF DATA FOR FIRST GRAPH.
C     NU = NO. OF UNITS /INCH ON X-AXIS.
C     XZERO = X-VALUE AT THE ORIGIN.
C     XEND = LARGEST VALUE OF THE ABSCISSA.
C     YZERO = Y-VALUE AT THE ORIGIN.
C     YEND = LARGEST VALUE OF THE ORDINATE.
C     XLG = LENGTH, IN INCHES, OF THE ABSCISSA.
C     YLG = LENGTH,IN INCHES,OF THE ORDINATES.
C     DELTA = FRACTION OF CYCLE DRAWN/INCH ON ORDINATE.
C     TYPE OF PLOT
C     PTYPE =1 FOR CARTESIAN PLOT
C           = 2 FOR SEMI-LOG PLOT, ORDINATE IS LOG AXIS
C     LABELX=X-COORDINATE VALUE WHERE LABEL BEGINS
C     LABELY=Y-COORDINATE VALUE WHERE LABEL BEGINS
C     IF LABELX & LABELY NOT GIVEN, LABEL BEGINS 2
C     INCHES FROM END OF PLOT & 2 INCHES FROM THE TOP.
C     IF PTYPE =1 NO VALUE IS NEEDED FOR DELTA
C     JPLOT.LT.0   POINT PLOT.
C     JPLOT.EQ.0   LINE PLOT
C     JPLOT.GT.0   POINT PLOT EVERY J-TH POINT
C     SYM(1) = SYMBOL USED FOR FIRST PLOT,IF ANY.
C     SYM(2) = SYMBOL USED FOR 2ND PLOT,IF ANY.
C     SYM(3) = SYMBOL USED FOR 3RD PLOT,IF ANY.
C     SYM(4) = SYMBOL USED FOR 4TH PLOT,IF ANY.
C     THIRD CARD   TX =ARRAY CONTAINING X-AXIS LABEL
C                  TY =ARRAY CONTAINING Y-AXIS LABEL
      READ 800,NG
  800 FORMAT(I1)
      KOUNT=0
      DO35NGPH=1,NG
      READ 801,NP,NU,XZERO,XEND,YZERO,YEND,XLG,YLG,DELTA,
     1PTYPE,JPLOT,SYM,TX,TY
  801 FORMAT(I1,I4,3F5.2,F10.2,3F5.0,6I2/8A8/8A8)
      KOUNT=KOUNT+1
      DO30NPLTS=1,NP
```

```
      KTER=0
C
C     LAST CARD OF EACH DATA SET MUST HAVE A * IN COL 80
C
      READ(5,807) LABEL,XABELX,YABELY
807   FORMAT(2A8,2F3.0)
      DO20I=1,5000
      READ(5,810,END= 25) X(I),Y(I),CHECK
      KTER=KTER+1
      IF(CHECK.EQ.XORE)GOTO25
```

```
   20 CONTINUE
  810 FORMAT(F10.0,E20.8,T80,A1)
   25 IND=XEND-XZERO
  250 IF(XABELX.LE.0.) XABELX=XLG-2.
      IF(YABELY.LE.0.) YABELY=YLG-2.
      IF(KTER.LE.301) GO TO 33
      PRINT 950,KTER
      STOP
   33 IF(NPLTS.LE.1) GO TO 26
   31 IF(PTYPE.EQ.1) GO TO 28
   32 IF(PTYPE.EQ.2) GO TO 29
   26 X(KTER+1)=XZERO
      X(KTER+2)=(XEND-XZERO)/XLG
      Y(KTER+1)=YZERO
      Y(KTER+2)=DELTA
      IF(PTYPE.EQ.1)Y(KTER+2)=(YEND-YZERO)/YLG
      PRINT 998,KTER
  998 FORMAT(1X,'K BEGIN PLOT',I6,'POINTS')
C     DRAW X AXIS & LABEL IT.
      CALL AXIS(0.0,0.0,FAKE,-4,XLG,0.,X(KTER+1),X(KTER+2))
      CALL PLOT(0.0,YLG,+3)
      CALL PLOT(XLG,YLG,+2)
      CALL SYMBOL(0.5,-0.5,0.14,TX ,0.,64)
C     DRAW Y AXIS AND LABEL IT
      IF(PTYPE.EQ.2)GOTO27
      CALL AXIS(0.0,0.0,FAKE,4,YLG,90.,YZERO,Y(KTER+2))
      CALL PLOT(XLG,0.0,+3)
      CALL PLOT(XLG,YLG,+2)
      CALL SYMBOL(-0.3,0.5,0.14,TY ,90.,64)
      CALL PLOT(0.0,0.0,+3)
   28 CALL LINE(X,Y,KTER,1,-1,3)
      CALL SYMBOL(XABELX,YABELY,0.14,LABEL,0.,16)
      GOTO30
   27 CALL LGAXIS(0.0,0.0,FAKE,4,YLG,90.,YZERO,DELTA)
      CALL PLOT(XLG,0.,+3)
      CALL PLOT(XLG,YLG,+2)
      CALL SYMBOL(-0.3,0.5,0.14,TY ,90.,64)
C     PLOT SCATTER
      CALL PLOT(0.0,0.0,+3)
   29 CALL LGLINE(X,Y,KTER,1,JPLOT,0,1)
      CALL SYMBOL(XABELX,YABELY,0.14,LABEL,0.,16)
   30 CONTINUE
      IF(NG.EQ.KOUNT)GOTO900
      XNEW=XLG+3.5
      CALL PLOT(XNEW,0.0,-3)
   35 CONTINUE
  950 FORMAT(1X,'MORE THAN ',I5,'DATA POINTS. '
     1'  INCREASE DIMENSION OF X&Y ARRAYS.')
```

```
900 PRINT 806
    CALL PLOT(0.,0.,999)
806 FORMAT(1H1,'JOB END')
    STOP
    END
```

APPENDIX III

```
$JOB          7011.50005,PAGES=20,TIME=180,LINES=50
C PROGRAM FOR CALCULATING THE ABSORPTION COEFFICIENT
C OF A HOMOGENIOUS MIXTURE,IN THOMSON UNIT/ELECTRON.
C GIVEN .......
C               1) WT.% COMPOSITION OF THE CONSTITUENTS.
C               2) AT. WEIGHT.
C               3) AT. NUMBER
C               4) MASS ABSORPTION COEFFICIENT OF EACH
C                  CONSTITUENT IN CM**2/GM.
C               5) EXCLUDE COHERENT SCATTERING.
C REFERENCES ......
C COMPOSITION OF CONCRETE
C ----------------------
C ENGINEERING COMPENDIUM ON RADIATION SHIELDING.VOL 1,
C PAGE 177,EDITED BY M.GROTENHUIS,A.HONIG,TH.A.JAEGER.
C MASS ABSORPTION COEFFICIENT DATA ON CONCRETE..
C ---------------------------------------------
C '' X-RAY ATTENUATION COEFFICIENTS FROM 10KEV TO
C 100MEV'',NBS REPORT(MICROFILM)1003,MAY13,1952
C THEORY..
C --------
C ''PENETRATION OF GAMMA ENERGY AT INFINITE MEDIUM'',
C NYO 3075,PP 62,GOLDSTEINE AND WILKINS.
C ---------------------------------------------------
C NOMENCLATURE ...
C ------------
C W(I) .......WT% COMPOSITION
C A(I) .......ATOMIC WT.
C Z(I) .......ATOMIC NUMBER
C E    .......ENERGY,MEV
C COEFFM.....MASS ABSORPTION COEFFICIENT OF CONCRETE
C IN (CM**2/GM)
C MUTHOM(I)...ABSORPTION COEFFICIENT OF EACH CONSTITU-
C ENT IN THOMSON UNIT/ELECTRON
C COEFFT......ABSORPTION COEFFICIENT OF CONCRETE IN
C THOMSON UNIT/ELECTRON
C ---------------------------------------------------
C COMPOSITION ,ATOMIC WT. AND ATOMIC NO.
C ------------------------------------
C HYDROGEN... 1) Z(1) .... 1.0
C             1) A(1) .... 1.008
C             1) W(1) .... 0.0056
C OXYGEN ... 2) Z(2) .... 8.0
C            2) A(2) ....16.0000
C            2) W(2) .... 0.4893
C SODIUM ... 3) Z(3) ....11.0
C            3) A(3) ....22.9900
C            3) W(3) .... 0.0171
C MAGNESIUM.. 4) Z(4) ....12.0
C             4) W(4) .... 0.0.0024
```

```
C ALUMINUM .. 5) Z(5) ....13.0
C             5) A(5) ....26.9800
C             5) W(5) .... 0.0456
C SILICON ... 6) Z(6) ....14.0
C             6) A(6) ....28.0900
C             6) W(6) .... 0.3158
C SULFUR   .. 7) Z(7) ....16.0
C             7) A(7) ....32.0600
C             7) W(7) .... 0.0012
C POTASSIUM . 8) Z(8) ....19.0
C             8) A(8) ....39.1000
C             8) W(8) .... 0.0192
C CALCIUM ... 9) Z(9) ....20.0
C             9) A(9) ....40.0800
C             9) W(9) .... 0.0826
C IRON ......10) Z(10)....26.0
C            10) A(10)....55.8500
C            10) W(10).... 0.0122
      REAL * 4 MUMASS,MUTHOM,LAMDA
      DIMENSION A(10),Z(10),W(10),MUMASS(10),MUTHOM(10)
     1,BETA(10)
      READ(5,10) (W(I),A(I),Z(I),I=1,10)
10    FORMAT(3F10.0)
      WRITE(6,65)
65    FORMAT(1H1,5X,' ABSORPTION COEFFICIENT DATA ON',
     1 'CONCRETE '/16X,' IN THOMSON UNIT/ELECTRON. '/)
15    READ(5,20) E,COEFFM
20    FORMAT(2F10.0)
      IF(COEFFM.EQ.0.) GO TO 80
      READ(5,30) (MUMASS(I),I=1,10)
30    FORMAT(10F7.0)
      SUM=0.
      DO 40 I=1,10
      SUM=SUM+(W(I)*Z(I))/A(I)
40    CONTINUE
      DO 50 I=1,10
      BETA(I)=(W(I)*Z(I)/A(I))/SUM
50    CONTINUE
      COEFFT=0.
      DO 60 I=1,10
      MUTHOM(I)=(MUMASS(I)*A(I))/(Z(I)*0.4005)
      COEFFT=COEFFT+MUTHOM(I)*BETA(I)
60    CONTINUE
      LAMDA=0.511/E
      WRITE(6,70) E,COEFFT,COEFFM,LAMDA
70    FORMAT(F10.4,5X,F9.4,5X,F9.4,5X,F10.4)
      GO TO 15
80    STOP
      END
```

ABSORPTION COEFFICIENT DATA ONCONCRETE
IN THOMSON UNIT/ELECTRON.

0.0100	132.1160	24.6000	51.1000
0.0200	17.2141	3.3400	25.5500
0.0300	5.5742	1.1000	17.0333
0.0400	2.7877	0.5420	12.7700
0.0500	1.8007	0.3500	10.2200
0.0600	1.3636	0.2670	8.5167
0.0700	1.0010	0.1970	6.3857
0.1000	0.8511	0.1690	5.1100
0.1500	0.6966	0.1390	3.4067
0.2000	0.6249	0.1240	2.5550
0.3000	0.5263	0.1070	1.7033
0.4000	0.4778	0.0954	1.2775
0.5000	0.4357	0.0870	1.0220
0.6000	0.4163	0.0804	0.8517
0.8000	0.3535	0.0706	0.6387
1.0000	0.3177	0.0635	0.5110
1.5000	0.2589	0.0517	0.3407
2.0000	0.2232	0.0445	0.2555
3.0000	0.1820	0.0363	0.1703
4.0000	0.1592	0.0317	0.1277
5.0000	0.1442	0.0290	0.1022
6.0000	0.1346	0.0270	0.0852
8.0000	0.1224	0.0245	0.0639
10.0000	0.1154	0.0231	0.0511

APPENDIX IV

ABSORPTION COEFFICIENT DATA ONCONCRETE
IN THOMSON UNIT/ELECTRON.

0.0100	132.1160	24.6000	51.1000
0.0200	17.2141	3.3400	25.5500
0.0300	5.5742	1.1000	17.0333
0.0400	2.7877	0.5420	12.7750
0.0500	1.8007	0.3500	10.2200
0.0600	1.3636	0.2670	8.5167
0.0800	1.0010	0.1970	6.3875
0.1000	0.8511	0.1690	5.1100
0.1500	0.6966	0.1390	3.4067
0.2000	0.6249	0.1240	2.5550
0.3000	0.5363	0.1070	1.7033
0.4000	0.4778	0.0954	1.2775
0.5000	0.4357	0.0870	1.0220
0.6000	0.4163	0.0804	0.8517
0.8000	0.3535	0.0706	0.6387
1.0000	0.3177	0.0635	0.5110
1.5000	0.2589	0.0517	0.3407
2.0000	0.2232	0.0445	0.2555
3.0000	0.1820	0.0363	0.1703
4.0000	0.1592	0.0317	0.1277
5.0000	0.1447	0.0290	0.1022
6.0000	0.1346	0.0270	0.0852
8.0000	0.1224	0.0245	0.0639
10.0000	0.1154	0.0231	0.0511

APPENDIX V

DEPTH OF PENETRATION 1.

ENERGY (MEV)	DIFFERENTIAL ANGULAR ENERGY FLUX
1.0000	0.12063322E 01
0.9274	0.11224642E 01
0.8646	0.10120249E 01
0.8098	0.95841122E 00
0.7616	0.91552544E 00
0.7187	0.88250732E 00
0.6804	0.83619595E 00
0.6460	0.81189251E 00
0.6149	0.79245853E 00
0.5867	0.77861786E 00
0.5609	0.76825523E 00
0.5373	0.77034187E 00
0.5156	0.76961803E 00
0.4956	0.77246666E 00
0.4771	0.77703667E 00
0.4599	0.78473186E 00
0.4440	0.77987099E 00
0.4291	0.78777599E 00
0.4151	0.79685783E 00
0.4020	0.80853462E 00
0.3898	0.82136059E 00
0.3782	0.83665085E 00
0.3674	0.85290051E 00
0.3571	0.87154675E 00
0.3474	0.87992191E 00
0.3382	0.89928913E 00
0.3295	0.91902161E 00
0.3212	0.94111633E 00
0.3133	0.96348190E 00
0.3058	0.98816586E 00
0.2987	0.10130482E 01
0.2918	0.10401239E 01
0.2853	0.10672712E 01
0.2791	0.10967293E 01
0.2731	0.11258831E 01
0.2674	0.11573601E 01
0.2619	0.11885109E 01
0.2567	0.12218962E 01
0.2516	0.12546244E 01
0.2467	0.12563915E 01
0.2421	0.12862091E 01
0.2376	0.13169947E 01
0.2332	0.13472795E 01
0.2290	0.13789768E 01
0.2250	0.14095078E 01
0.2211	0.14417496E 01
0.2174	0.14726248E 01

0.2137	0.15049553E 01
0.2102	0.15356779E 01
0.2068	0.15676575E 01
0.2035	0.15978632E 01
0.2003	0.13069878E 01
0.1972	0.13324947E 01
0.1942	0.13523798E 01
0.1913	0.13734894E 01
0.1885	0.13917952E 01
0.1858	0.14117680E 01
0.1831	0.14285698E 01
0.1805	0.14469652E 01
0.1780	0.14626751E 01
0.1755	0.14795322E 01
0.1732	0.14652052E 01
0.1708	0.14778976E 01
0.1686	0.14878130E 01
0.1664	0.14991102E 01
0.1643	0.15082493E 01
0.1622	0.15189390E 01
0.1601	0.15275822E 01
0.1582	0.15375557E 01
0.1562	0.15458355E 01
0.1543	0.15554409E 01
0.1525	0.15632381E 01
0.1507	0.15722828E 01
0.1489	0.15798969E 01
0.1472	0.15889587E 01
0.1455	0.15963717E 01
0.1439	0.16048746E 01
0.1423	0.16120138E 01
0.1407	0.16202536E 01
0.1392	0.16269836E 01
0.1377	0.16347895E 01
0.1362	0.16407642E 01
0.1348	0.16477470E 01
0.1334	0.16528149E 01
0.1320	0.16585655E 01
0.1307	0.16625690E 01
0.1293	0.16666269E 01
0.1280	0.16686668E 01
0.1268	0.16709909E 01
0.1255	0.16712456E 01
0.1243	0.15903816E 01
0.1231	0.15832996E 01
0.1219	0.15735321E 01
0.1208	0.15620985E 01
0.1196	0.15483809E 01
0.1185	0.15324221E 01
0.1174	0.15138884E 01
0.1164	0.14926691E 01
0.1153	0.14685831E 01
0.1143	0.14413338E 01

0.1133	0.14721088E 01
0.1123	0.14520416E 01
0.1113	0.14844646E 01
0.1103	0.14891882E 01
0.1094	0.16035595E 01
0.1085	0.16662750E 01
0.1076	0.18443031E 01
0.1067	0.19648552E 01
0.1058	0.21639338E 01
0.1049	0.22249556E 01
0.1041	0.22810612E 01
0.1032	0.22269630E 01
0.1024	0.22651091E 01
0.1016	0.22114611E 01
0.1008	0.22421484E 01
0.1000	0.21891041E 01
0.0992	0.22135649E 01
0.0984	0.21609316E 01
0.0977	0.21797581E 01
0.0969	0.21275558E 01
0.0962	0.21416063E 01
0.0955	0.20897722E 01
0.0948	0.21000595E 01
0.0941	0.20483780E 01
0.0934	0.20554790E 01
0.0927	0.20045252E 01
0.0921	0.20088482E 01
0.0914	0.19585552E 01
0.0907	0.19610958E 01
0.0901	0.19109974E 01
0.0895	0.18330841E 01
0.0889	0.17811604E 01
0.0882	0.17760515E 01
0.0876	0.17249699E 01
0.0870	0.17197094E 01
0.0864	0.16693783E 01
0.0859	0.16661816E 01
0.0853	0.16163988E 01
0.0847	0.16153622E 01
0.0842	0.15663347E 01
0.0836	0.15684357E 01
0.0831	0.15201902E 01
0.0825	0.15259848E 01
0.0820	0.14783707E 01
0.0815	0.14891567E 01
0.0810	0.14420328E 01
0.0805	0.14578266E 01
0.0800	0.14111452E 01
0.0795	0.14326410E 01
0.0790	0.13864498E 01
0.0785	0.14557705E 01

DOSE 0.20856885E-06 BUILD UP FACTOR 0.19580116E 01

DEPTH OF PENETRATION 2.

ENERGY (MEV)	DIFFERENTIAL ANGULAR ENERGY FLUX
1.0000	0.24126530E 01
0.9274	0.23046818E 01
0.8646	0.20461750E 01
0.8098	0.19796305E 01
0.7616	0.19249506E 01
0.7187	0.18916187E 01
0.6804	0.17861309E 01
0.6460	0.17550488E 01
0.6149	0.17272243E 01
0.5867	0.17144279E 01
0.5609	0.17033062E 01
0.5373	0.17359009E 01
0.5156	0.17482662E 01
0.4956	0.17717743E 01
0.4771	0.17936878E 01
0.4599	0.18254089E 01
0.4440	0.18005323E 01
0.4291	0.18234215E 01
0.4151	0.18421879E 01
0.4020	0.18706827E 01
0.3898	0.18955021E 01
0.3782	0.19294653E 01
0.3674	0.19596605E 01
0.3571	0.19985552E 01
0.3474	0.19954071E 01
0.3382	0.20309563E 01
0.3295	0.20607634E 01
0.3212	0.20996466E 01
0.3133	0.21332464E 01
0.3058	0.21756716E 01
0.2987	0.22127380E 01
0.2918	0.22584553E 01
0.2853	0.22987194E 01
0.2791	0.23475609E 01
0.2731	0.23908081E 01
0.2674	0.24426136E 01
0.2619	0.24886761E 01
0.2567	0.25433226E 01
0.2516	0.25920334E 01
0.2467	0.25672092E 01
0.2421	0.26088600E 01
0.2376	0.26550322E 01
0.2332	0.26973381E 01
0.2290	0.27456379E 01
0.2250	0.27895727E 01
0.2211	0.28398619E 01
0.2174	0.28854218E 01

0.2137	0.29375486E 01
0.2102	0.29846029E 01
0.2068	0.30384436E 01
0.2035	0.30868931E 01
0.2003	0.28905535E 01
0.1972	0.29689302E 01
0.1942	0.30452614E 01
0.1913	0.31180067E 01
0.1885	0.31892395E 01
0.1858	0.32585554E 01
0.1831	0.33257313E 01
0.1805	0.33906536E 01
0.1780	0.34535551E 01
0.1755	0.35140381E 01
0.1732	0.34875536E 01
0.1708	0.35343580E 01
0.1686	0.35758495E 01
0.1664	0.36174440E 01
0.1643	0.36556854E 01
0.1622	0.36949549E 01
0.1601	0.37308550E 01
0.1582	0.37675343E 01
0.1562	0.38012009E 01
0.1543	0.38356867E 01
0.1525	0.38673325E 01
0.1507	0.38997564E 01
0.1489	0.39299593E 01
0.1472	0.39614172E 01
0.1455	0.39904518E 01
0.1439	0.40204763E 01
0.1423	0.40481091E 01
0.1407	0.40766802E 01
0.1392	0.41027336E 01
0.1377	0.41296301E 01
0.1362	0.41537657E 01
0.1348	0.41784916E 01
0.1334	0.42001791E 01
0.1320	0.42221527E 01
0.1307	0.42406912E 01
0.1293	0.42590256E 01
0.1280	0.42733183E 01
0.1268	0.42882080E 01
0.1255	0.42992086E 01
0.1243	0.40611191E 01
0.1231	0.40525923E 01
0.1219	0.40342607E 01
0.1208	0.40159960E 01
0.1196	0.39905148E 01
0.1185	0.39637117E 01
0.1174	0.39294958E 01
0.1164	0.38926516E 01
0.1153	0.38480768E 01
0.1143	0.37995672E 01

0.1133	0.38690481E 01
0.1123	0.38589220E 01
0.1113	0.39931564E 01
0.1103	0.40704660E 01
0.1094	0.44938583E 01
0.1095	0.47533026E 01
0.1076	0.53744774E 01
0.1067	0.57880774E 01
0.1058	0.64716644E 01
0.1049	0.66774054E 01
0.1041	0.69082012E 01
0.1032	0.67429075E 01
0.1024	0.69192429E 01
0.1016	0.67556524E 01
0.1008	0.69054899E 01
0.1000	0.67435923E 01
0.0992	0.68691740E 01
0.0984	0.67088985E 01
0.0977	0.68126955E 01
0.0969	0.66541014E 01
0.0962	0.67387371E 01
0.0955	0.65817347E 01
0.0948	0.66498384E 01
0.0941	0.64943752E 01
0.0934	0.65485468E 01
0.0927	0.63945484E 01
0.0921	0.64373713E 01
0.0914	0.62848806E 01
0.0907	0.63190660E 01
0.0901	0.61679354E 01
0.0895	0.58818254E 01
0.0889	0.57231989E 01
0.0882	0.57150717E 01
0.0876	0.55596218E 01
0.0870	0.55482349E 01
0.0864	0.53958330E 01
0.0859	0.53864260E 01
0.0853	0.52368383E 01
0.0847	0.52318525E 01
0.0842	0.50849218E 01
0.0836	0.50865870E 01
0.0831	0.49421453E 01
0.0825	0.49527617E 01
0.0820	0.48106308E 01
0.0815	0.48322506E 01
0.0810	0.46922283E 01
0.0805	0.47268324E 01
0.0800	0.45887775E 01
0.0795	0.46382856E 01
0.0790	0.45020542E 01
0.0785	0.46867714E 01

DOSE 0.30743681E-07 BUILD UP FACTOR 0.31381683E 01

DEPTH OF PENETRATION 4.

ENERGY (MEV)	DIFFERENTIAL ANGULAR ENERGY FLUX
1.0000	0.48252869E 01
0.9274	0.48490915E 01
0.8646	0.41796312E 01
0.8098	0.41980524E 01
0.7616	0.41825666E 01
0.7187	0.42316647E 01
0.6804	0.39540644E 01
0.6460	0.39503651E 01
0.6149	0.39122925E 01
0.5867	0.39321442E 01
0.5609	0.39236851E 01
0.5373	0.40779715E 01
0.5156	0.41312494E 01
0.4956	0.42342491E 01
0.4771	0.43053694E 01
0.4599	0.44204245E 01
0.4440	0.43128395E 01
0.4291	0.43821030E 01
0.4151	0.44114027E 01
0.4020	0.44855890E 01
0.3898	0.45253191E 01
0.3782	0.46066208E 01
0.3674	0.46554403E 01
0.3571	0.47435617E 01
0.3474	0.46837950E 01
0.3382	0.47552156E 01
0.3295	0.47902575E 01
0.3212	0.48653250E 01
0.3133	0.49077177E 01
0.3058	0.49884882E 01
0.2987	0.50378723E 01
0.2918	0.51245899E 01
0.2853	0.51809559E 01
0.2791	0.52737761E 01
0.2731	0.53373213E 01
0.2674	0.54366970E 01
0.2619	0.55074711E 01
0.2567	0.56138668E 01
0.2516	0.56923904E 01
0.2467	0.56055832E 01
0.2421	0.56702881E 01
0.2376	0.57582226E 01
0.2332	0.58270617E 01
0.2290	0.59231796E 01
0.2250	0.60001345E 01
0.2211	0.61050787E 01
0.2174	0.61905308E 01

0.2137	0.12326777E 02
0.2102	0.12479170E 02
0.2068	0.12720325E 02
0.2035	0.12887988E 02
0.2003	0.13045273E 02
0.1972	0.13479753E 02
0.1942	0.13940785E 02
0.1913	0.14361430E 02
0.1885	0.14796003E 02
0.1858	0.15206729E 02
0.1831	0.15623662E 02
0.1805	0.16015488E 02
0.1780	0.16412201E 02
0.1755	0.16782974E 02
0.1732	0.16678757E 02
0.1708	0.16969116E 02
0.1686	0.17234344E 02
0.1664	0.17491684E 02
0.1643	0.17736984E 02
0.1622	0.17981583E 02
0.1601	0.18212570E 02
0.1582	0.18441284E 02
0.1562	0.18657654E 02
0.1543	0.18872482E 02
0.1525	0.19075363E 02
0.1507	0.19276657E 02
0.1489	0.19469543E 02
0.1472	0.19664719E 02
0.1455	0.19849792E 02
0.1439	0.20035645E 02
0.1423	0.20211761E 02
0.1407	0.20388962E 02
0.1392	0.20555511E 02
0.1377	0.20723251E 02
0.1362	0.20878952E 02
0.1348	0.21035126E 02
0.1334	0.21178711E 02
0.1320	0.21321594E 02
0.1307	0.21449997E 02
0.1293	0.21575714E 02
0.1280	0.21684296E 02
0.1268	0.21796082E 02
0.1255	0.21892059E 02
0.1243	0.20553665E 02
0.1231	0.20543457E 02
0.1219	0.20468704E 02
0.1208	0.20410156E 02
0.1196	0.20307220E 02
0.1185	0.20212601E 02
0.1174	0.20073792E 02
0.1164	0.19935623E 02
0.1153	0.19753021E 02
0.1143	0.19562607E 02

0.1133	0.19879028E 02
0.1123	0.19977081E 02
0.1113	0.20841476E 02
0.1103	0.21473633E 02
0.1094	0.24112244E 02
0.1085	0.25807083E 02
0.1076	0.29608627E 02
0.1067	0.32119492E 02
0.1058	0.36293015E 02
0.1049	0.37523453E 02
0.1041	0.39080673E 02
0.1032	0.38139145E 02
0.1024	0.39400513E 02
0.1016	0.38468704E 02
0.1008	0.39558228E 02
0.1000	0.38637085E 02
0.0992	0.39567505E 02
0.0984	0.38656479E 02
0.0977	0.39444031E 02
0.0969	0.38542633E 02
0.0962	0.39201477E 02
0.0955	0.38310593E 02
0.0948	0.38856415E 02
0.0941	0.37975159E 02
0.0934	0.38423492E 02
0.0927	0.37551743E 02
0.0921	0.37918823E 02
0.0914	0.37056778E 02
0.0907	0.37358246E 02
0.0901	0.36504944E 02
0.0895	0.34655304E 02
0.0889	0.33751221E 02
0.0882	0.33733429E 02
0.0876	0.32850586E 02
0.0870	0.32795822E 02
0.0864	0.31932434E 02
0.0859	0.31877899E 02
0.0853	0.31033432E 02
0.0847	0.30992950E 02
0.0842	0.30165833E 02
0.0836	0.30153610E 02
0.0831	0.29343185E 02
0.0825	0.29371552E 02
0.0820	0.28576202E 02
0.0815	0.28658417E 02
0.0810	0.27877060E 02
0.0805	0.28023132E 02
0.0800	0.27255157E 02
0.0795	0.27475845E 02
0.0790	0.26719650E 02
0.0785	0.27670776E 02

DOSE 0.57653229E-10 BUILD UP FACTOR 0.10699224E 02

DEPTH OF PENETRATION 7.

ENERGY (MEV)	DIFFERENTIAL ANGULAR ENERGY FLUX
1.0000	0.84443064E 01
0.9274	0.91190777E 01
0.8646	0.75394468E 01
0.8098	0.79284010E 01
0.7616	0.80426903E 01
0.7187	0.83804274E 01
0.6804	0.76882324E 01
0.6460	0.78013144E 01
0.6149	0.77113962E 01
0.5867	0.78298492E 01
0.5609	0.77924662E 01
0.5373	0.82226467E 01
0.5156	0.83245316E 01
0.4956	0.86089993E 01
0.4771	0.87473717E 01
0.4599	0.90464897E 01
0.4440	0.87381687E 01
0.4291	0.89120169E 01
0.4151	0.89297304E 01
0.4020	0.91024075E 01
0.3898	0.91422653E 01
0.3782	0.93244371E 01
0.3674	0.93840485E 01
0.3571	0.95765276E 01
0.3474	0.93950148E 01
0.3382	0.95475616E 01
0.3295	0.95763378E 01
0.3212	0.97338610E 01
0.3133	0.97802486E 01
0.3058	0.99486256E 01
0.2987	0.10011918E 02
0.2918	0.10191939E 02
0.2853	0.10271664E 02
0.2791	0.10464365E 02
0.2731	0.10560243E 02
0.2674	0.10766247E 02
0.2619	0.10878588E 02
0.2567	0.11098693E 02
0.2516	0.11227381E 02
0.2467	0.11051183E 02
0.2421	0.11152374E 02
0.2376	0.11330824E 02
0.2332	0.11439188E 02
0.2290	0.11632892E 02
0.2250	0.11755343E 02
0.2211	0.11964489E 02
0.2174	0.12101830E 02

0.2137	0.63051577E 01
0.2102	0.63999958E 01
0.2068	0.65251245E 01
0.2035	0.66302137E 01
0.2003	0.65713978E 01
0.1972	0.67809544E 01
0.1942	0.70017176E 01
0.1913	0.72053080E 01
0.1885	0.74137669E 01
0.1858	0.76110401E 01
0.1831	0.78104677E 01
0.1805	0.79978857E 01
0.1780	0.81866112E 01
0.1755	0.83633137E 01
0.1732	0.83099670E 01
0.1708	0.84482098E 01
0.1686	0.85742502E 01
0.1664	0.86968727E 01
0.1643	0.88134050E 01
0.1622	0.89295588E 01
0.1601	0.90390396E 01
0.1582	0.91476450E 01
0.1562	0.92499533E 01
0.1543	0.93517151E 01
0.1525	0.94477501E 01
0.1507	0.95431538E 01
0.1489	0.96343546E 01
0.1472	0.97265406E 01
0.1455	0.98138962E 01
0.1439	0.99019642E 01
0.1423	0.99850054E 01
0.1407	0.10068615E 02
0.1392	0.10147142E 02
0.1377	0.10225973E 02
0.1362	0.10299441E 02
0.1348	0.10372910E 02
0.1334	0.10440202E 02
0.1320	0.10507108E 02
0.1307	0.10566539E 02
0.1293	0.10625175E 02
0.1280	0.10675011E 02
0.1268	0.10726427E 02
0.1255	0.10769760E 02
0.1243	0.10124543E 02
0.1231	0.10116757E 02
0.1219	0.10079185E 02
0.1208	0.10047844E 02
0.1196	0.99956827E 01
0.1185	0.99461546E 01
0.1174	0.98756933E 01
0.1164	0.98038025E 01
0.1153	0.97104559E 01
0.1143	0.96114435E 01

0.1133	0.97709923E 01
0.1123	0.98053722E 01
0.1113	0.10214775E 02
0.1103	0.10504603E 02
0.1094	0.11759098E 02
0.1085	0.12558776E 02
0.1076	0.14368798E 02
0.1067	0.15566205E 02
0.1058	0.17551727E 02
0.1049	0.18140915E 02
0.1041	0.18866547E 02
0.1032	0.18412827E 02
0.1024	0.18993530E 02
0.1016	0.18544525E 02
0.1008	0.19044998E 02
0.1000	0.18600601E 02
0.0992	0.19026810E 02
0.0984	0.18587326E 02
0.0977	0.18946243E 02
0.0969	0.18511597E 02
0.0962	0.18810669E 02
0.0955	0.18380829E 02
0.0948	0.18627365E 02
0.0941	0.18202240E 02
0.0934	0.18403748E 02
0.0927	0.17983200E 02
0.0921	0.18147385E 02
0.0914	0.17731064E 02
0.0907	0.17865295E 02
0.0901	0.17453369E 02
0.0895	0.16586792E 02
0.0889	0.16151077E 02
0.0882	0.16139862E 02
0.0876	0.15714001E 02
0.0870	0.15687230E 02
0.0864	0.15270737E 02
0.0859	0.15244957E 02
0.0853	0.14837187E 02
0.0847	0.14819344E 02
0.0842	0.14419755E 02
0.0836	0.14416312E 02
0.0831	0.14024293E 02
0.0825	0.14041449E 02
0.0820	0.13656545E 02
0.0815	0.13699743E 02
0.0810	0.13321533E 02
0.0805	0.13396636E 02
0.0800	0.13024610E 02
0.0795	0.13136744E 02
0.0790	0.12770333E 02
0.0785	0.13238160E 02

DOSE 0.19592949E-08 BUILD UP FACTOR 0.59111137E 01

DEPTH OF PENETRATION 10.

ENERGY (MEV)	DIFFERENTIAL ANGULAR ENERGY FLUX
1.0000	0.12063189E 02
0.9274	0.13938766E 02
0.8646	0.11087960E 02
0.8098	0.12098132E 02
0.7616	0.12334103E 02
0.7187	0.13112516E 02
0.6804	0.11821227E 02
0.6460	0.12121024E 02
0.6149	0.11906319E 02
0.5867	0.12169325E 02
0.5609	0.12045548E 02
0.5373	0.12818576E 02
0.5156	0.12930140E 02
0.4956	0.13446782E 02
0.4771	0.13624736E 02
0.4599	0.14157858E 02
0.4440	0.13596611E 02
0.4291	0.13913723E 02
0.4151	0.13894025E 02
0.4020	0.14202085E 02
0.3898	0.14222186E 02
0.3782	0.14541090E 02
0.3674	0.14596345E 02
0.3571	0.14928438E 02
0.3474	0.14598705E 02
0.3382	0.14862955E 02
0.3295	0.14872018E 02
0.3212	0.15140883E 02
0.3133	0.15179253E 02
0.3058	0.15463103E 02
0.2987	0.15528770E 02
0.2918	0.15829344E 02
0.2853	0.15921277E 02
0.2791	0.16239365E 02
0.2731	0.16357239E 02
0.2674	0.16695068E 02
0.2619	0.16838226E 02
0.2567	0.17196945E 02
0.2516	0.17366013E 02
0.2467	0.17099640E 02
0.2421	0.17226379E 02
0.2376	0.17516815E 02
0.2332	0.17655029E 02
0.2290	0.17969055E 02
0.2250	0.18131027E 02
0.2211	0.18470123E 02
0.2174	0.18656143E 02

0.2137	0.19021820E 02
0.2102	0.19233124E 02
0.2068	0.19627380E 02
0.2035	0.19865768E 02
0.2003	0.20275009E 02
0.1972	0.20964203E 02
0.1942	0.21696381E 02
0.1913	0.22360214E 02
0.1885	0.23049286E 02
0.1858	0.23698044E 02
0.1831	0.24361298E 02
0.1805	0.24981018E 02
0.1780	0.25611832E 02
0.1755	0.26199127E 02
0.1732	0.26040802E 02
0.1708	0.26500961E 02
0.1686	0.26922409E 02
0.1664	0.27330093E 02
0.1643	0.27720718E 02
0.1622	0.28108932E 02
0.1601	0.28476761E 02
0.1582	0.28840439E 02
0.1562	0.29184982E 02
0.1543	0.29525696E 02
0.1525	0.29849442E 02
0.1507	0.30169037E 02
0.1489	0.30476608E 02
0.1472	0.30786026E 02
0.1455	0.31081055E 02
0.1439	0.31376602E 02
0.1423	0.31657211E 02
0.1407	0.31938004E 02
0.1392	0.32204071E 02
0.1377	0.32469101E 02
0.1362	0.32718155E 02
0.1348	0.32966187E 02
0.1334	0.33195450E 02
0.1320	0.33422440E 02
0.1307	0.33626617E 02
0.1293	0.33827499E 02
0.1280	0.34001129E 02
0.1268	0.34179520E 02
0.1255	0.34333862E 02
0.1243	0.32221512E 02
0.1231	0.32209015E 02
0.1219	0.32092911E 02
0.1208	0.32004318E 02
0.1196	0.31844421E 02
0.1185	0.31699692E 02
0.1174	0.31484543E 02
0.1164	0.31271881E 02
0.1153	0.30988403E 02
0.1143	0.30694946E 02

0.1133	0.31186646E 02
0.1123	0.31355118E 02
0.1113	0.32727890E 02
0.1103	0.33742691E 02
0.1094	0.37924210E 02
0.1085	0.40614777E 02
0.1076	0.46630142E 02
0.1067	0.50600876E 02
0.1058	0.57198822E 02
0.1049	0.59138519E 02
0.1041	0.61611145E 02
0.1032	0.60126678E 02
0.1024	0.62130875E 02
0.1016	0.60662186E 02
0.1008	0.62395523E 02
0.1000	0.60942215E 02
0.0992	0.62424347E 02
0.0984	0.60987579E 02
0.0977	0.62241104E 02
0.0969	0.60820175E 02
0.0962	0.61870361E 02
0.0955	0.60465332E 02
0.0948	0.61336151E 02
0.0941	0.59946732E 02
0.0934	0.60663269E 02
0.0927	0.59289139E 02
0.0921	0.59875854E 02
0.0914	0.58516083E 02
0.0907	0.58998978E 02
0.0901	0.57653809E 02
0.0895	0.54723801E 02
0.0889	0.53297470E 02
0.0882	0.53270935E 02
0.0876	0.51877945E 02
0.0870	0.51793259E 02
0.0864	0.50432404E 02
0.0859	0.50346603E 02
0.0853	0.49015076E 02
0.0847	0.48951508E 02
0.0842	0.47648270E 02
0.0836	0.47627167E 02
0.0831	0.46349548E 02
0.0825	0.46393799E 02
0.0820	0.45140259E 02
0.0815	0.45265793E 02
0.0810	0.44034286E 02
0.0805	0.44261795E 02
0.0800	0.43051666E 02
0.0795	0.43396576E 02
0.0790	0.42205948E 02
0.0785	0.43699158E 02

DOSE 0.21054495E-11 BUILD UP FACTOR 0.16016251E 02

DEPTH OF PENETRATION 15.

ENERGY (MEV)	DIFFERENTIAL ANGULAR ENERGY FLUX
1.0000	0.18094772E 02
0.9274	0.23213821E 02
0.8646	0.17423706E 02
0.8098	0.19942871E 02
0.7616	0.20213379E 02
0.7187	0.21986176E 02
0.6804	0.19348969E 02
0.6460	0.20098373E 02
0.6149	0.19517395E 02
0.5867	0.20121506E 02
0.5609	0.19738358E 02
0.5373	0.21196930E 02
0.5156	0.21238251E 02
0.4956	0.22239380E 02
0.4771	0.22416031E 02
0.4599	0.23432297E 02
0.4440	0.22361954E 02
0.4291	0.22992859E 02
0.4151	0.22858383E 02
0.4020	0.23459290E 02
0.3898	0.23401154E 02
0.3782	0.24011246E 02
0.3674	0.24018692E 02
0.3571	0.24643631E 02
0.3474	0.24009933E 02
0.3382	0.24514618E 02
0.3295	0.24453598E 02
0.3212	0.24960785E 02
0.3133	0.24952423E 02
0.3058	0.25480118E 02
0.2987	0.25521805E 02
0.2918	0.26074036E 02
0.2853	0.26161575E 02
0.2791	0.26743210E 02
0.2731	0.26875092E 02
0.2674	0.27486847E 02
0.2619	0.27664063E 02
0.2567	0.28309662E 02
0.2516	0.28530289E 02
0.2467	0.28137146E 02
0.2421	0.28290024E 02
0.2376	0.28817368E 02
0.2332	0.28988342E 02
0.2290	0.29553757E 02
0.2250	0.29760880E 02
0.2211	0.30364334E 02
0.2174	0.30610275E 02

0.2137	0.31253098E 02
0.2102	0.31537460E 02
0.2068	0.32219650E 02
0.2035	0.32542740E 02
0.2003	0.33428223E 02
0.1972	0.34574173E 02
0.1942	0.35793442E 02
0.1913	0.36892181E 02
0.1885	0.38036636E 02
0.1858	0.39117371E 02
0.1831	0.40217545E 02
0.1805	0.41251511E 02
0.1780	0.42302414E 02
0.1755	0.43282257E 02
0.1732	0.43020889E 02
0.1708	0.43789688E 02
0.1686	0.44495087E 02
0.1664	0.45175705E 02
0.1643	0.45828674E 02
0.1622	0.46477066E 02
0.1601	0.47092941E 02
0.1582	0.47699478E 02
0.1562	0.48276779E 02
0.1543	0.48847610E 02
0.1525	0.49388184E 02
0.1507	0.49923523E 02
0.1489	0.50437744E 02
0.1472	0.50957260E 02
0.1455	0.51451385E 02
0.1439	0.51944672E 02
0.1423	0.52415405E 02
0.1407	0.52886002E 02
0.1392	0.53331039E 02
0.1377	0.53777054E 02
0.1362	0.54193481E 02
0.1348	0.54609772E 02
0.1334	0.54992554E 02
0.1320	0.55372574E 02
0.1307	0.55717926E 02
0.1293	0.56052094E 02
0.1280	0.56346451E 02
0.1268	0.56644363E 02
0.1255	0.56904984E 02
0.1243	0.53391190E 02
0.1231	0.53371918E 02
0.1219	0.53180252E 02
0.1208	0.53034653E 02
0.1196	0.52770432E 02
0.1185	0.52532791E 02
0.1174	0.52176849E 02
0.1164	0.51828751E 02
0.1153	0.51361496E 02
0.1143	0.50877777E 02

0.1133	0.51687210E 02
0.1123	0.51983215E 02
0.1113	0.54275116E 02
0.1103	0.55978729E 02
0.1094	0.62954163E 02
0.1085	0.67449799E 02
0.1076	0.77478897E 02
0.1067	0.84096863E 02
0.1058	0.95097076E 02
0.1049	0.98326981E 02
0.1041	0.10245831E 03
0.1032	0.99988525E 02
0.1024	0.10335262E 03
0.1016	0.10090910E 03
0.1008	0.10381577E 03
0.1000	0.10139940E 03
0.0992	0.10388626E 03
0.0984	0.10149648E 03
0.0977	0.10360330E 03
0.0969	0.10124040E 03
0.0962	0.10300633E 03
0.0955	0.10066922E 03
0.0948	0.10213416E 03
0.0941	0.99822403E 02
0.0934	0.10102972E 03
0.0927	0.98743790E 02
0.0921	0.99733246E 02
0.0914	0.97473770E 02
0.0907	0.98284698E 02
0.0901	0.96049347E 02
0.0895	0.91148911E 02
0.0889	0.88777740E 02
0.0882	0.88738281E 02
0.0876	0.86422729E 02
0.0870	0.86280396E 02
0.0864	0.84016174E 02
0.0859	0.83873764E 02
0.0853	0.81659882E 02
0.0847	0.81551010E 02
0.0842	0.79383713E 02
0.0836	0.79347488E 02
0.0831	0.77223892E 02
0.0825	0.77288666E 02
0.0820	0.75206039E 02
0.0815	0.75415573E 02
0.0810	0.73370392E 02
0.0805	0.73741913E 02
0.0800	0.71731445E 02
0.0795	0.72296204E 02
0.0790	0.70317551E 02
0.0785	0.72790405E 02

DOSE 0.10135152E-13 BUILD UP FACTOR 0.25745514E 02

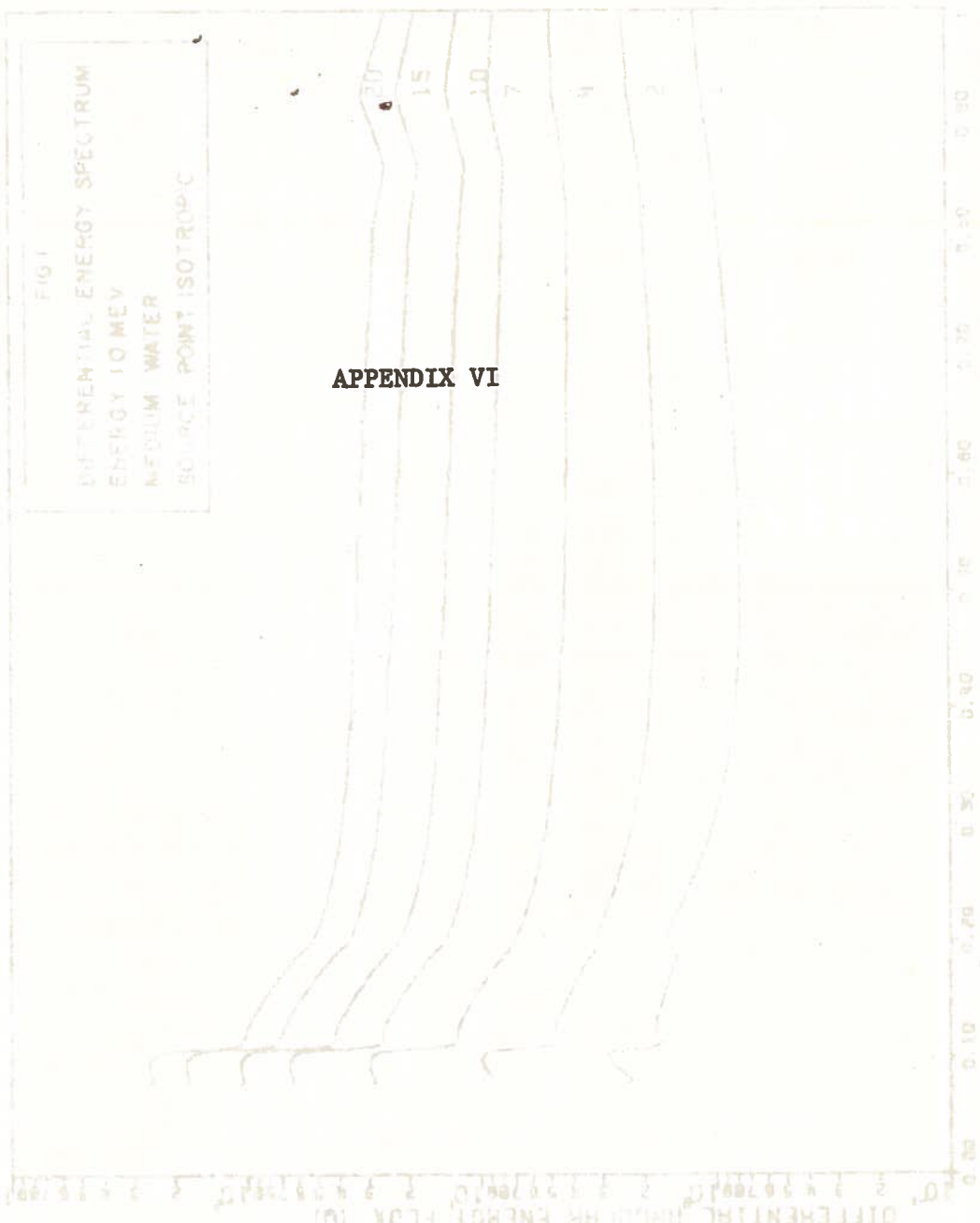
DEPTH OF PENETRATION 20.

ENERGY (MEV)	DIFFERENTIAL ANGULAR ENERGY FLUX
1.0000	0.24125366E 02
0.9274	0.34072784E 02
0.8646	0.24299545E 02
0.8098	0.28854782E 02
0.7616	0.28852936E 02
0.7187	0.31906250E 02
0.6804	0.27572189E 02
0.6460	0.28936554E 02
0.6149	0.27815811E 02
0.5867	0.28891678E 02
0.5609	0.28120483E 02
0.5373	0.30402969E 02
0.5156	0.30279175E 02
0.4956	0.31876053E 02
0.4771	0.31973709E 02
0.4599	0.33576721E 02
0.4440	0.31893692E 02
0.4291	0.32918259E 02
0.4151	0.32600174E 02
0.4020	0.33571030E 02
0.3898	0.33375275E 02
0.3782	0.34352295E 02
0.3674	0.34260986E 02
0.3571	0.35253418E 02
0.3474	0.34253662E 02
0.3382	0.35064209E 02
0.3295	0.34893555E 02
0.3212	0.35703125E 02
0.3133	0.35614746E 02
0.3058	0.36452637E 02
0.2987	0.36436035E 02
0.2918	0.37307129E 02
0.2853	0.37362061E 02
0.2791	0.38268311E 02
0.2731	0.38389648E 02
0.2674	0.39338379E 02
0.2619	0.39522705E 02
0.2567	0.40515869E 02
0.2516	0.40762451E 02
0.2467	0.40264160E 02
0.2421	0.40413330E 02
0.2376	0.41228027E 02
0.2332	0.41402100E 02
0.2290	0.42265625E 02
0.2250	0.42492432E 02
0.2211	0.43410889E 02
0.2174	0.43687256E 02

0.2137	0.44659912E 02
0.2102	0.44987549E 02
0.2068	0.46020752E 02
0.2035	0.46399658E 02
0.2003	0.47843994E 02
0.1972	0.49481445E 02
0.1942	0.51218750E 02
0.1913	0.52780762E 02
0.1885	0.54410400E 02
0.1858	0.55945557E 02
0.1831	0.57524414E 02
0.1805	0.58990234E 02
0.1780	0.60492920E 02
0.1755	0.61884766E 02
0.1732	0.61515625E 02
0.1708	0.62607178E 02
0.1686	0.63612305E 02
0.1664	0.64579834E 02
0.1643	0.65510254E 02
0.1622	0.66434570E 02
0.1601	0.67311523E 02
0.1582	0.68178223E 02
0.1562	0.69001221E 02
0.1543	0.69815918E 02
0.1525	0.70590820E 02
0.1507	0.71348389E 02
0.1489	0.72088867E 02
0.1472	0.72823486E 02
0.1455	0.73528320E 02
0.1439	0.74232422E 02
0.1423	0.74902344E 02
0.1407	0.75574707E 02
0.1392	0.76208252E 02
0.1377	0.76843262E 02
0.1362	0.77438965E 02
0.1348	0.78031738E 02
0.1334	0.78582764E 02
0.1320	0.79127686E 02
0.1307	0.79615234E 02
0.1293	0.80097168E 02
0.1280	0.80508545E 02
0.1268	0.80937012E 02
0.1255	0.81309570E 02
0.1243	0.76287109E 02
0.1231	0.76264160E 02
0.1219	0.75986816E 02
0.1208	0.75777588E 02
0.1196	0.75400146E 02
0.1185	0.75057861E 02
0.1174	0.74551025E 02
0.1164	0.74048096E 02
0.1153	0.73376221E 02
0.1143	0.72691406E 02

0.1133	0.73846924E 02
0.1123	0.74258789E 02
0.1113	0.77524170E 02
0.1103	0.79949463E 02
0.1094	0.89893066E 02
0.1085	0.96294922E 02
0.1076	0.11059155E 03
0.1067	0.12002368E 03
0.1058	0.13570508E 03
0.1049	0.14029077E 03
0.1041	0.14619067E 03
0.1032	0.14266870E 03
0.1024	0.14745020E 03
0.1016	0.14396338E 03
0.1008	0.14810107E 03
0.1000	0.14465405E 03
0.0992	0.14819653E 03
0.0984	0.14478320E 03
0.0977	0.14777881E 03
0.0969	0.14439990E 03
0.0962	0.14690991E 03
0.0955	0.14357544E 03
0.0948	0.14566479E 03
0.0941	0.14236426E 03
0.0934	0.14407813E 03
0.0927	0.14082104E 03
0.0921	0.14222119E 03
0.0914	0.13899487E 03
0.0907	0.14015747E 03
0.0901	0.13696899E 03
0.0895	0.12998901E 03
0.0889	0.12659644E 03
0.0882	0.12653540E 03
0.0876	0.12323047E 03
0.0870	0.12303394E 03
0.0864	0.11980127E 03
0.0859	0.11959644E 03
0.0853	0.11643994E 03
0.0847	0.11628247E 03
0.0842	0.11319360E 03
0.0836	0.11314233E 03
0.0831	0.11010938E 03
0.0825	0.11021558E 03
0.0820	0.10724194E 03
0.0815	0.10752637E 03
0.0810	0.10460693E 03
0.0805	0.10514819E 03
0.0800	0.10227759E 03
0.0795	0.10309399E 03
0.0790	0.10027417E 03
0.0785	0.10380591E 03

DOSE 0.54371376E-16 BUILD UP FACTOR 0.36441086E 02



APPENDIX VI

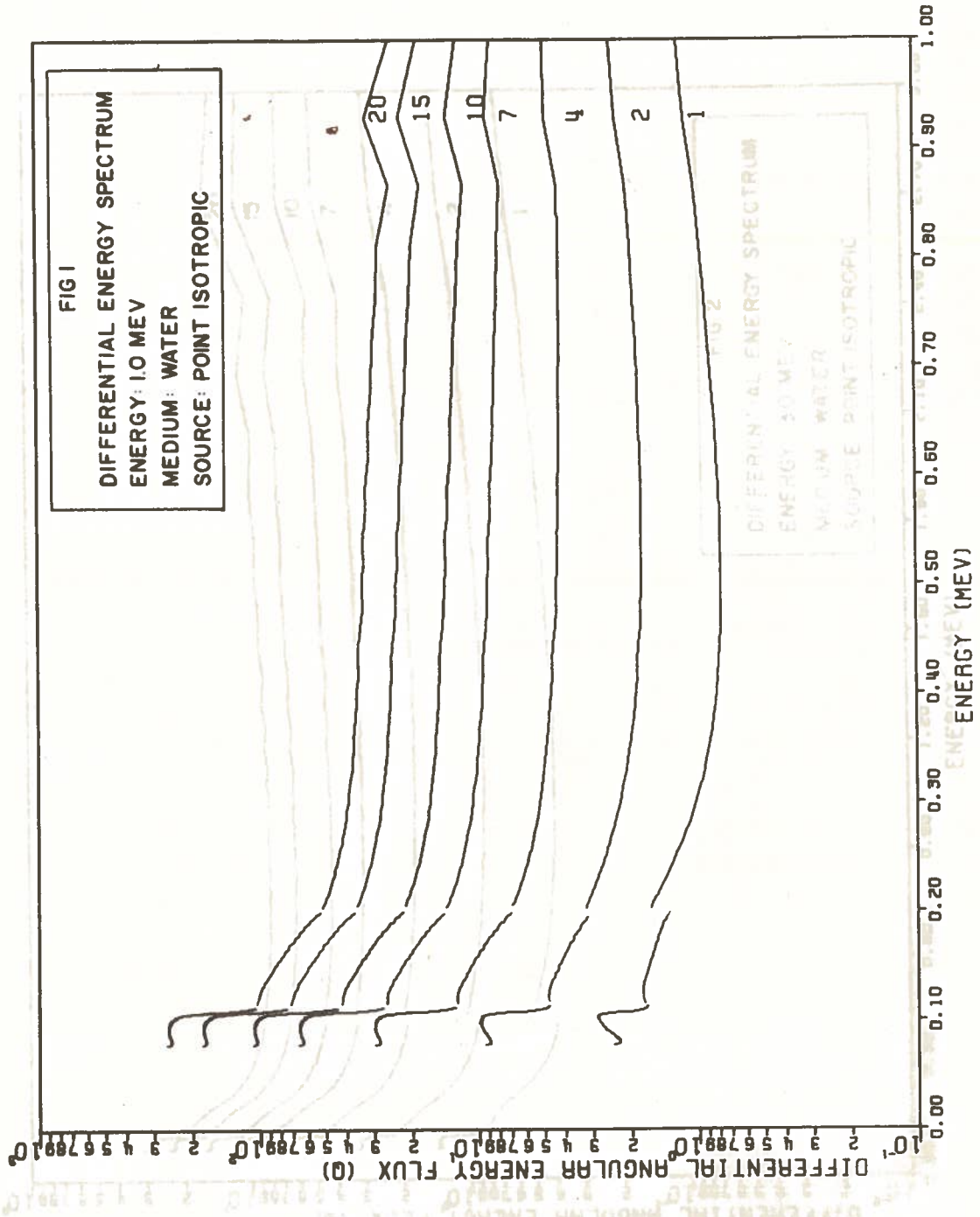


FIG I
DIFFERENTIAL ENERGY SPECTRUM
ENERGY: 1.0 MEV
MEDIUM: WATER
SOURCE: POINT ISOTROPIC
20
15
10
7
4
2
1
DIFFERENTIAL ANGULAR ENERGY FLUX (Q)
ENERGY (MEV)
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00

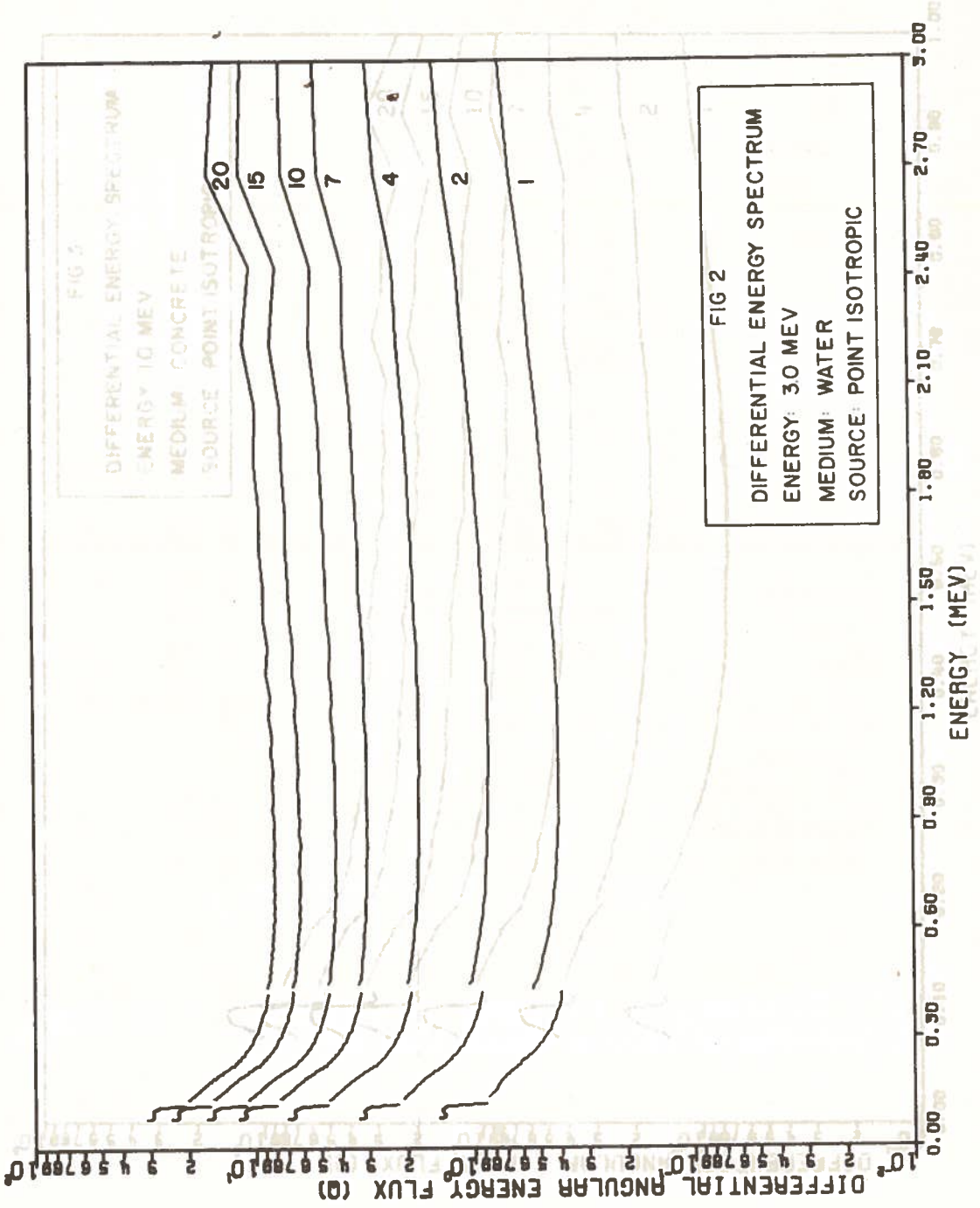


FIG 2
DIFFERENTIAL ENERGY SPECTRUM
ENERGY: 30 MEV
MEDIUM: WATER
SOURCE: POINT ISOTROPIC
20
15
10
7
4
2
1
0.00
0.30
0.60
0.80
1.20
1.50
1.80
2.10
2.40
2.70
3.00
ENERGY (MEV)
DIFFERENTIAL ANGULAR ENERGY FLUX (Q)

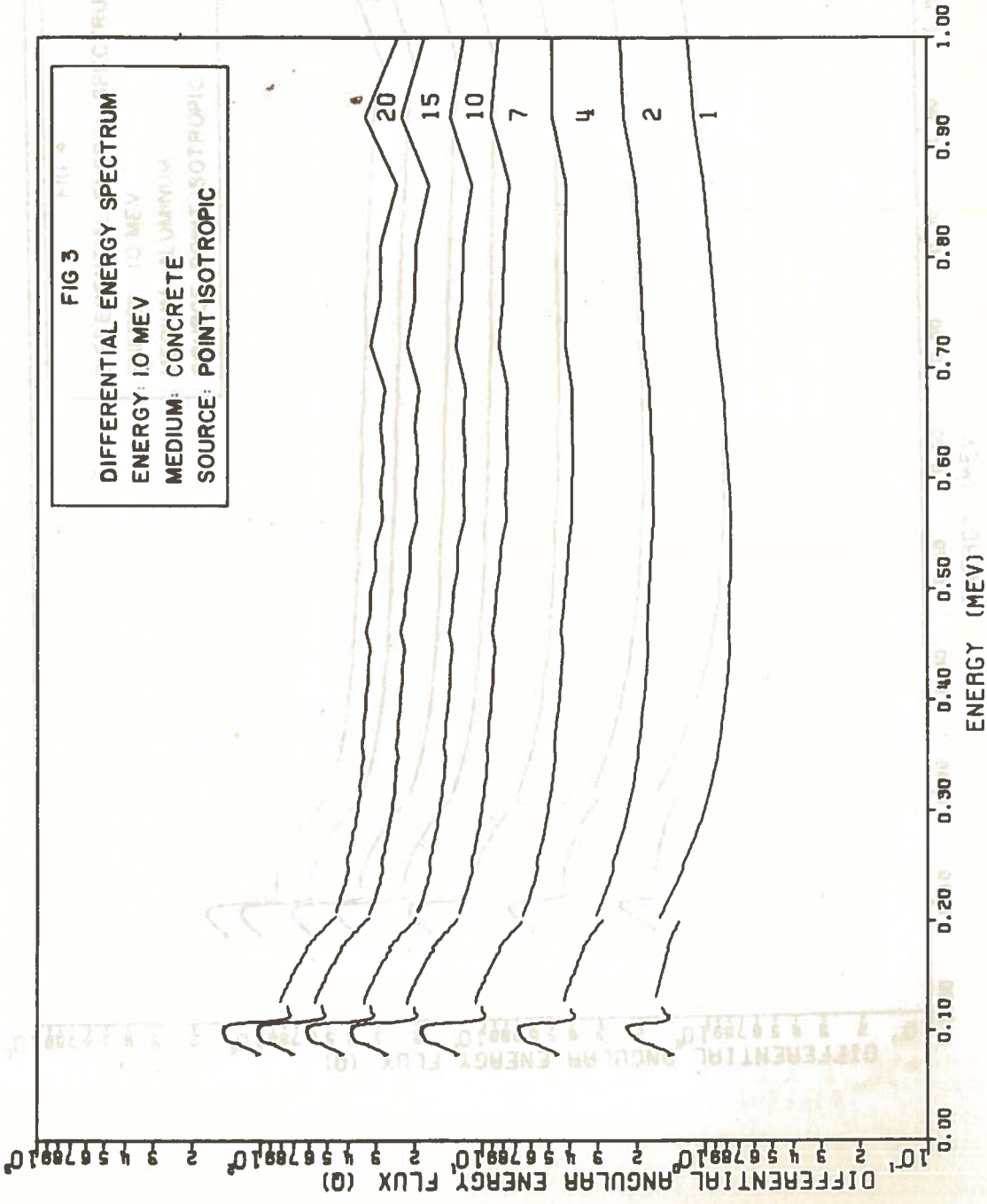


FIG 3
DIFFERENTIAL ENERGY SPECTRUM
ENERGY: 10 MEV
MEDIUM: CONCRETE
SOURCE: POINT ISOTROPIC
20
15
10
7
4
2
1
DIFFERENTIAL ANGULAR ENERGY FLUX (Q)
ENERGY (MEV)
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00

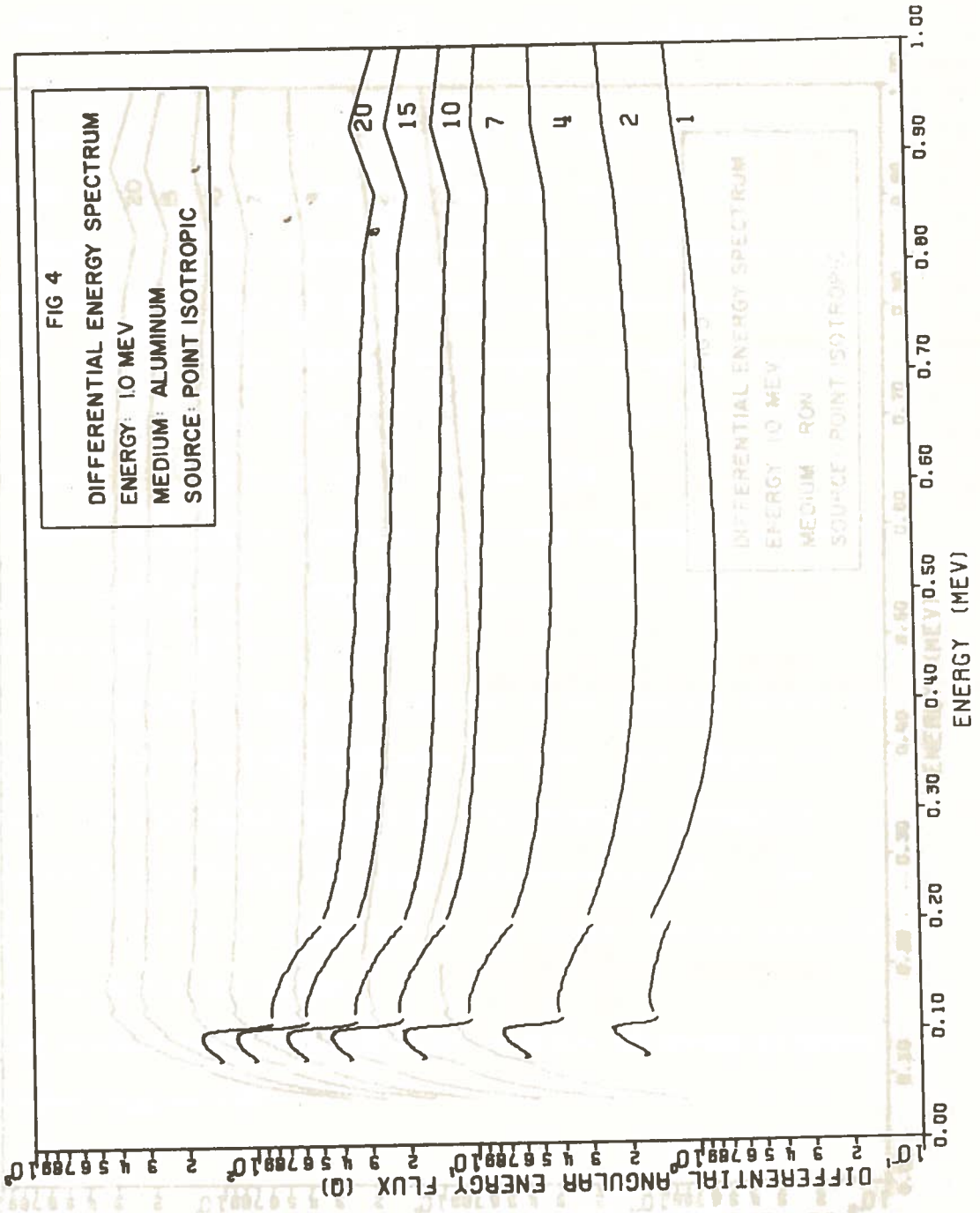


FIG 4
DIFFERENTIAL ENERGY SPECTRUM
ENERGY: 1.0 MEV
MEDIUM: ALUMINUM
SOURCE: POINT ISOTROPIC
20
15
10
7
4
2
1
DIFFERENTIAL ANGULAR ENERGY FLUX (Ω)
ENERGY (MEV)
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00

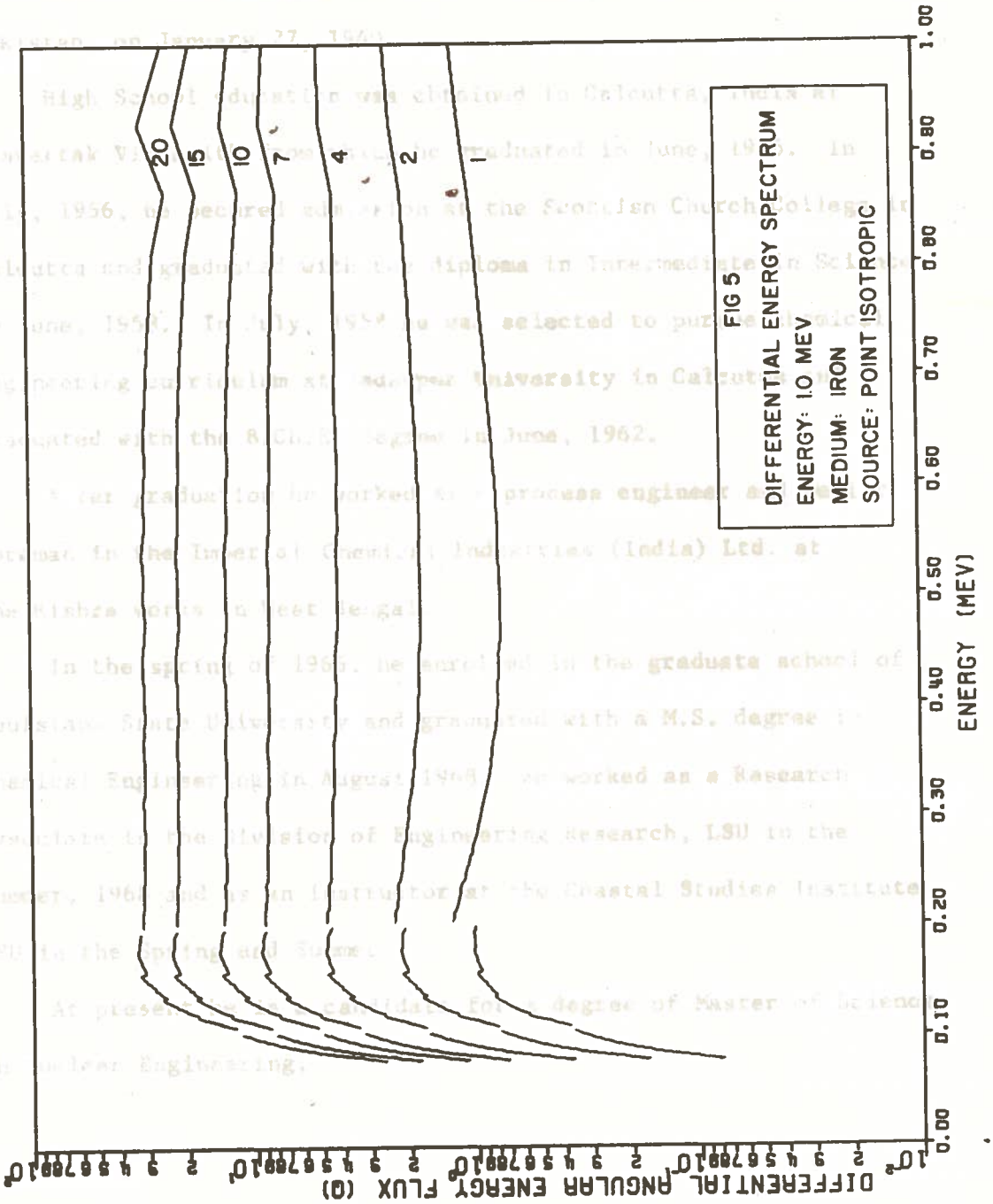


FIG 5
DIFFERENTIAL ENERGY SPECTRUM
ENERGY: 10 MEV
MEDIUM: IRON
SOURCE: POINT ISOTROPIC
20
15
10
7
4
2
1
ENERGY (MEV)
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
DIFFERENTIAL ANGULAR ENERGY FLUX (Q)

VITA

Arun P. Bhattacharya was born in Gouripur, Mymensingh, E. Pakistan, on January 27, 1940.

High School education was obtained in Calcutta, India at Pravartak Vidyapith from which he graduated in June, 1956. In July, 1956, he secured admission at the Scottish Church College in Calcutta and graduated with the diploma in Intermediate in Science in June, 1958. In July, 1958 he was selected to pursue chemical engineering curriculum at Jadavpur University in Calcutta and graduated with the B.Ch.E. degree in June, 1962.

After graduation he worked as a process engineer and Senior Foreman in the Imperial Chemical Industries (India) Ltd. at the Rishra works in West Bengal.

In the spring of 1966, he enrolled in the graduate school of Louisiana State University and graduated with a M.S. degree in Chemical Engineering in August 1968. He worked as a Research Associate in the Division of Engineering Research, LSU in the Summer, 1968 and as an instructor at the Coastal Studies Institute LSU in the Spring and Summer.

At present he is a candidate for a degree of Master of Science in Nuclear Engineering.

EXAMINATION AND THESIS REPORT

Candidate: Arun P. Bhattacharya

Major Field: Nuclear Engineering

Title of Thesis: Gamma Spectral Calculations by the Moments Method

Approved:

Major Professor and Chairman

Dean of the Graduate School

EXAMINING COMMITTEE:

Date of Examination:

April 28, 1971