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ORIGINAL ARTICLE

Mean–variance scaling and stability in commercial sex work networks

Tad A. Dallas[1] · Bret D. Elderd[2]

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Abstract

Understanding how networks change over time can help identify network properties related to stability and uncover general scaling rules of network evolution. In the analysis of static networks, the tendency of a network to form into *modules* has received the most support as a measure of network structure and potential stability signature. However, *modularity* and stability relationships have largely been explored only for networks of information flow (e.g., computer networks) and some bipartite interaction networks (e.g., plant-pollinator networks). We use a collection of over 80 commercial sex worker networks sampled across 18 years to explore the conservation of modularity over time. If modularity is a sign of stability in social or sexual networks, then modular networks should tend to stay modular over time, resulting in a negative mean–variance scaling. We find evidence of positive mean–variance scaling in network size, but a negative mean–variance scaling relationship for modularity. This suggests that commercial sex work networks conserve modularity over time, despite high turnover in the individual nodes (i.e., clients and sex workers) which make up the network. Together, our results link network structure and temporal network dynamics, and provide evidence for clear mean–variance scaling relationships in complex networks.

Keywords Sexual contact network · Commercial sex work · Network stability · Modularity · Network dynamics

1 Introduction

Networks are a great tool for visualizing and studying the complex interactions between individuals and between species (Tylianakis and Morris 2017). Social contact networks allow the examination of social group formation, cohesion, and opinion spreading (Backstrom et al. 2006; Ndeffo Mbah et al. 2012). In particular, sexual contact networks—which link individuals in networks based on their sexual interactions—can be used to identify interacting groups, and explore potential epidemic dynamics (Meyers et al. 2006). However, data on sexual contact networks tend to be limited due to their potentially sensitive nature. However, the rise of digital platforms that allow sex workers to interact with clients, and clients to review sex workers, has created available time-resolved sexual interaction data. These data are incredibly important to understanding the structure of interaction networks more generally, as they represent a clear instance of contact network through which dynamic processes occur such as infectious disease propagation (Robinson et al. 2012) and opinion spreading (Xie et al. 2012). In the case of sexual contact networks, sex workers and clients represent nodes in a network, with links based on sexual encounters between sex workers and clients. While we do not attempt to claim that commercial sex work networks are somehow representative of sexual contact networks generally, they are often the closest we can estimate, as data on empirical sexual contact networks tend to be small in scale, size, and temporal resolution.

Sex worker-client networks may be examined as static, where interaction order and time are largely not considered (Hsieh et al. 2014), but both sex workers and clients may only be present in the network for a short time, suggesting that a lot can be gained by addressing the temporal component explicitly (Rocha et al. 2010). For instance, dividing networks into snapshots in time allows for the estimation of both mean and variance of network properties and their relationship (e.g., do they positively covary?). This scaling between mean and variance is found in a variety of systems, and generally tends

✉ Tad A. Dallas
tdallas@mailbox.sc.edu

[1] Department of Biological Sciences, University of South Carolina, Columbia, SC 29208, USA

[2] Department of Biological Sciences, Louisiana State University, Baton Rouge, LA 70803, USA

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to be positive. Evidence for positive mean–variance scaling relationships comes from dynamics of animal and plant populations (Reuman et al. 2017), species cell counts (Azevedo and Leroi 2001), stock market activity (Cai et al. 2016), and internet traffic (Duch and Arenas 2006). Together, they suggest that the higher the mean value of something (e.g., individuals in a population), the more this value tends to fluctuate over time. For ecological systems, this leads to the idea that large populations should also be more variable, something at odds with population dynamic theory related to demographic stochasticity (Xiao et al. 2015), which shows that small populations are more variable.

The study of networks (i.e., graph theory) proposes a collection of hypotheses about how networks may be structured, and how network structure may translate into dynamic processes that determine stability. For instance, graph theory posits that networks arranged into subgroups (or modules), where individuals within subgroups interact mostly with nodes within their own subgroup, tend to be more resistant to perturbations. Until recently, this idea received theoretical support (Newman 2012; Grilli et al. 2016), but quite limited empirical tests (but see Gilarranz et al. (2017)). Support for modularity being a signature of network stability has been previously posited for ecological (Thébault and Fontaine 2010; Grilli et al. 2016; Gilarranz et al. 2017), social (Sales-Pardo et al. 2007), and technological (Duch and Arenas 2006) networks. If more modular networks tend to be more stable, we might expect that networks with a given modularity value in one time would have a similar modularity value in the next time, creating a negative mean–variance scaling relationship. This would be counter to the vast majority of positive mean–variance scaling relationships. Modularity is also related to pathogen invasion potential (Sah et al. 2017), where more modular networks tend to have smaller epidemics, depending on the disease model (Nadini et al. 2018). Thus, sustained modularity could be a signal of a stable network which is resistant to both epidemics and general perturbations (Gilarranz et al. 2017). More generally, the tendency for a network to retain a structural property over time is a signature of a stable network (Yoon et al. 2018). This suggests that a stable network should have relatively larger mean values of modularity, and that these networks with high mean modularity should also have small variance in modularity over time.

One of the largest stumbling blocks to the empirical evaluation of these types of relationships in sex worker networks—and networks more generally—is the paucity of available temporally resolved data, as most data on sexual contact networks come from surveys of small populations (Liljeros et al. 2001). Data on commercial sex work networks from websites which catalog sex worker—client interactions are temporally resolved, creating an incredibly rich data source from which to examine temporal changes to network properties and how structural features of networks relate to network dynamics. We explore how network properties change over time, leveraging data on 81 commercial sex work networks sampled over 18 years as part of *The Erotic Review*, an online platform for advertising and ranking commercial sex work. Specifically, we examine the scaling relationship between mean and variance in network size, connectance, and modularity. Network size and connectance were expected to demonstrate a positive mean–variance scaling relationship similar to other mean–variance scaling relationships. However, modularity may be a signature of network stability, suggesting that more modular networks are also more stable over time. Thus, we would expect a negative relationship between mean and variance in modularity, as more modular networks would tend to stay modular. We find clear positive mean–variance scaling relationships in network size (larger networks are more variable over time), and connectance (well-connected networks are more variable over time), but a negative mean–variance relationships for modularity (more modular networks are more consistently modular over time).

2 Methods

2.1 The Erotic Review data

The Erotic Review is an online platform which facilitates interactions between sex workers, who post a public profile with information, and clients, who provide reviews of their interactions with sex workers. *The Erotic Review* has no public API, and data were acquired by programmatically scraping data on each sex worker and associated reviews, anonymizing the sex worker and client usernames as we scraped. In total, we gathered data from 2000–2017 from *The Erotic Review* at the US county scale, selecting counties which housed the top 100 more populous US cities. We selected 2017 as the end year due to the USA passing legislation which shut down *The Erotic Review* in 2018. Overall, we had data from a total of 81 US counties, constituting over 980,000 unique sex worker–client interactions. These data provided putative client–sex worker associations, as client-reported reviews may suffer from under-reporting (not every sexual contact is likely reviewed) or over-reporting (disconnect between client-reported sexual encounter and reality). These reviews provide the basis of the sexual contact network, and fraudulent reviews can be flagged and moderated by website administrators. While biases in reporting interactions may exist, it is unlikely to vary systematically across US counties or through time, suggesting that temporal comparisons within the same county remain valid, and variation in network structure across counties is highly unlikely to be driven by differential reporting bias across space. Online sex worker–client networks represent an interesting way to explore temporal variation in network structure, and, if representative of more general sexual contact networks, are an indispensable resource for understanding other types of sexual contact networks. We built commercial sex work networks at the annual scale, as the data are timestamped to month, but this corresponds to the time of review, not the time of interaction. An edge between two nodes in a network represents a review, which we assume implies a previous sexual contact. In our networks, edges can only occur between a sex worker and a client, not between two sex workers or two clients. This created a total of 18 annual bipartite networks per US county. Both sex workers and clients can exist in more than a single network. These internetwork connections are interesting, but were not considered in this analysis, as the focus here is on the structural and temporal properties of individual networks.

2.2 Estimating network structure and modularity

For each client–sex worker network and each year, we estimated several properties, including network size, connectance, and modularity. Network size was estimated as the total number of sex workers and clients in the network. Connectance was estimated as the total fraction of realized associations in each network out of the all possible links (the number of sex workers times the number of clients). Finally, we estimated modularity using Barber's Q statistic, after delineating modules using the random walk method of Pons and Latapy (Pons and Latapy 2005). While we use modularity, or more accurately the scaling between the mean and variance of modularity, as a potential indicator of network stability, we acknowledge that other measures of network structural stability exist. For instance, the dominant eigenvalue of the adjacency matrix of a network may be a sign of structural stability (Song and Saavedra 2020). This measure is also a proposed measure of nestedness in networks as well (Staniczenko et al. 2013), a network property which is directly related to modularity (Fortuna et al. 2010).

Modularity inherently depends on the structure of the underlying network, such networks can vary in terms of maximum achievable modularity as a function of their connectance and network size. To account for this, we standardized modularity estimates based on the maximum modularity, which we calculated by simulating modular networks of the same size and number of links as the empirical network. To do this, we created 1000 simulated networks for each possible number of modules in the network, which ranged from 1 module to each node in its own module. Links within modules were randomly removed until the connectance of the empirical network was reached.

For each network, consisting of the client–sex worker network of a given county in a given year, we calculate these network statistics, as well as aggregate estimates of the mean and standard deviation for each county, allowing each county to exist as a single point in mean–variance network structure space. For a given county, the client–sex worker network may change over time. Larger, more established networks, may fluctuate less over time, resulting in a negative relationship between network size and the variance in network size.

2.3 Mean–variance relationships

With respect to modularity, a large body of theory would suggest that more modular networks should remain modular through time, as modularity is a potential indicator of *stability*, resulting in a negative mean–variance relationship (Thébault and Fontaine 2010; Grilli et al. 2016; Gilarranz et al. 2017; Sales-Pardo et al. 2007; Duch and Arenas 2006). To assess mean–variance scaling relationships when covariance among network size, connectance, and modularity may exist, we calculated relationship between mean and variance in network structure as partial correlation coefficients, which attempt to parse out the effects of collinear covariates to estimate the underlying relationships.

Larger networks may fluctuate more over time, as a function of the burstiness of popularity of the site in a given location, or driven by demographic or social factors surrounding sex work in that area. Commercial sex worker networks may also change in their membership over time, as the average indoor sex worker is active on *The Erotic Review* website for less than 2 years. This degree of turnover in membership, along with changes in client demand, creates a situation where we would expect the networks to be quite dynamic. This turnover may be related to stability, as measured as modularity. To explore this, we quantified mean turnover of sex workers and clients combined from year to year, and related this to both network size and modularity. We would expect that larger networks are more established, and generally have higher stability and lower turnover. We measure turnover here as

$$1 - \frac{|A \cap B|}{|A \cup B|}$$

where A and B correspond to the sex workers and clients for two different years. This is a modified Jaccard index, where larger values indicate larger compositional differences of sex workers and clients between two years.

R code and data to reproduce the analyses is provided at https://doi.org/10.6084/m9.figshare.14186663.v1.

3 Results

3.1 Commercial sex worker networks

Sex worker networks were broadly distributed over the USA (Fig. 1a), and varied markedly in time-averaged network size, with average network size ranging from 8 to nearly 5000 sex workers and clients. Networks also varied in structure, with connectance tending to be low but variable and modularity (standardized by maximum achievable modularity) being quite high, but variable (Fig. 1b). Finally, sex worker networks tended to increase in size over time (Fig. 1c).

3.2 Mean–variance scaling in network structure

Network size and the number of links in a network may fundamentally constrain higher-order properties of networks, such as modularity (Poisot and Gravel 2014). To explore a mean–variance scaling relationship in modularity, we must consider the relationships between lower-order network statistics and resulting modularity. By using partial correlations, we attempted to parse out the influence of network size and connectance on resulting mean–variance relationship in modularity (Fig. 2). We found positive mean–variance scaling relationships for network size (Fig. S1) and connectance (Fig. S2). Further, larger networks tended to have lower connectance (Fig. 2). Mean connectance was negatively related to network size and modularity, while the variation in connectance followed an opposing pattern (Fig. 2). Perhaps most importantly, we observed a clear negative relationship ($r = -0.51$, $p < 0.001$) between mean and variance (measured as standard deviation) in modularity (Figs. 2, 3).

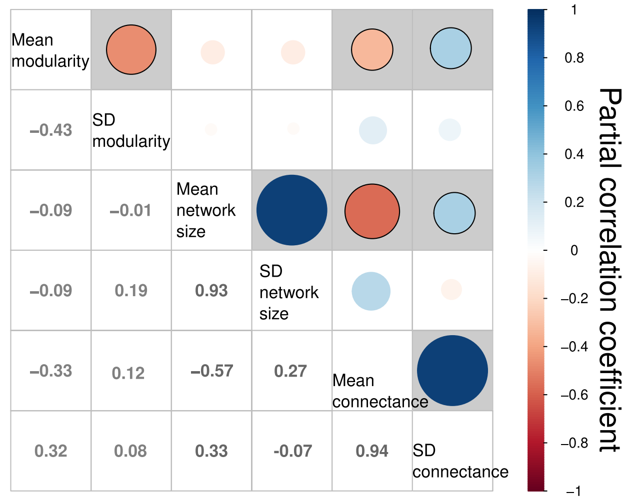

Fig. 2 Partial correlation coefficients between mean and variance measures in network structure, specifically in modularity, network size, and connectance. While positive mean–variance scaling relationships were observed for network size and connectance, a clear negative relationship was observed for modularity. Gray box outlines indicate statistical significance ($\alpha = 0.01$). Other weaker relationships existed between average network measures (e.g., connectance was negatively related to network size and modularity)

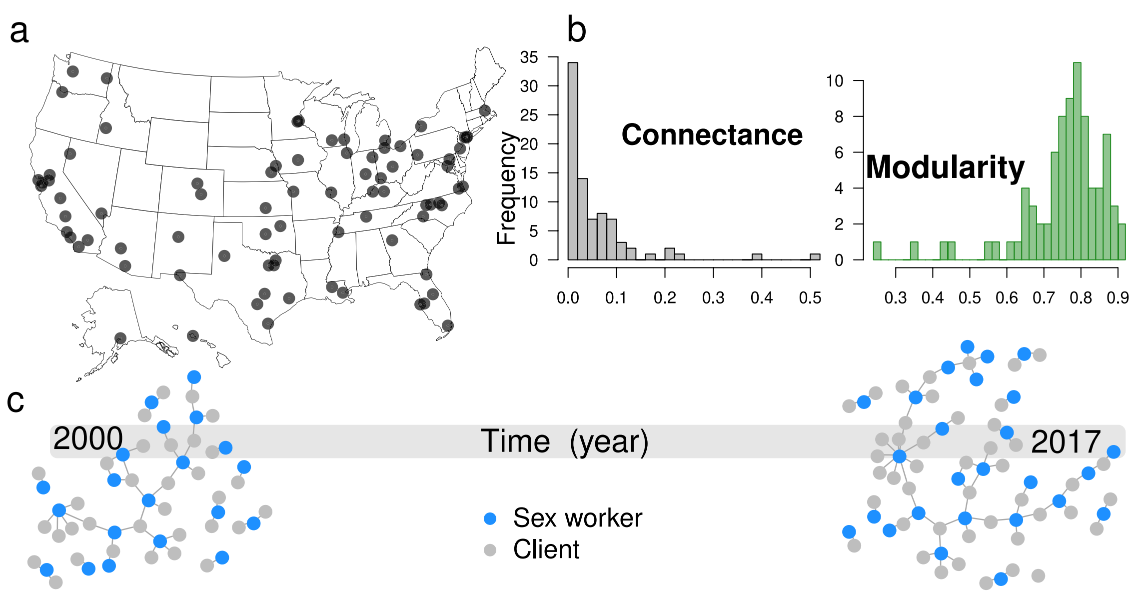

Fig. 1 Data on the interactions between clients and sex workers from *The Erotic Review* considered here were widespread across the USA (**a**) and varied greatly in average network properties such as connectance and modularity (**b**), with each network changing in structure over time (**c**)

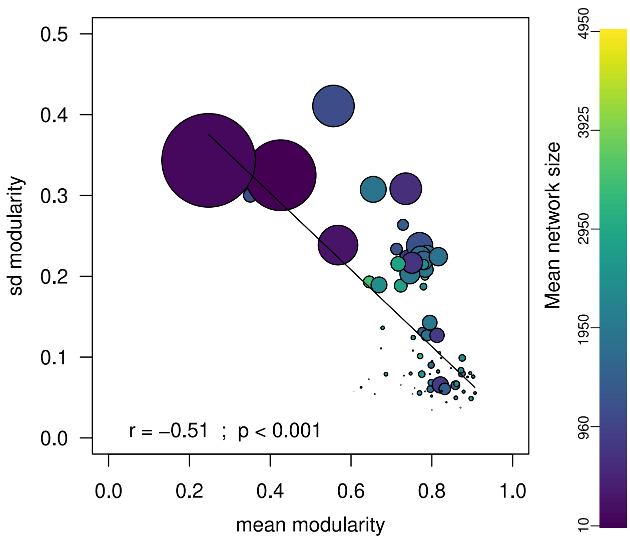

Fig. 3 Mean–variance scaling relationship in network modularity, where point color is proportional to average network size (the total number of sex workers and clients in the network) and point size is proportional to connectance (the fraction of realized links in a network). Smaller networks tended to also have higher connectance. Modular networks tended to have lower variance in that modularity, suggesting that modular networks stay modular. Reported correlations are Spearman's rank correlation coefficient (*r*) and associated *p* value. Plotted line is from a linear model, intended simply to highlight the relationship

3.3 Turnover in commercial sex worker networks

Turnover tended to be quite high on average, where over 90% of clients and sex workers were not shared between any two years. However, we observed no relationship—or a weak positive relationship—between modularity and turnover (Fig. 4). However, we do observe an effect of network size on mean turnover, suggesting that larger networks tend to have lower turnover (Fig. 4).

4 Discussion

In temporally resolved sex worker networks, we find clear evidence for positive mean–variance relationships for both network size and connectance. However, modularity had a negative mean–variance relationship, where higher mean modularity corresponded to less variance, suggesting that more modular networks tend to stay modular over time. The positive mean–variance relationships observed for network size and connectance suggest that larger networks are also more variable, just as more well-connected networks tend to be more variable over time. As modularity has been posited to be a signature of stable networks, the maintenance of this network property provides empirical evidence that more modular sexual contact networks tend to conserve modularity over time. We find this signature of network temporal stability even given the high turnover observed in the composition of sex workers and clients which make up the networks, suggesting some functional redundancy in sex workers and clients, as new nodes to the network can fill the same role as those leaving the network.

But what does *stability* in the context of a sexual contact network actually mean? It does not mean that individuals in the network play a consistent role over time, as individual sex workers or clients are only active for a short time in the networks. This level of turnover observed suggests that these networks are highly dynamic. The turnover observed, though, was unrelated to modularity, though it scaled with network size, suggesting larger networks have less turnover. Stable networks are those that have consistent temporal properties, such that different individuals play the same functional role through time. That is, even though turnover was fairly high, stability may be unaffected if the new individuals in the network play similar roles to those leaving the network. This disambiguation from lower-level properties of individual nodes to higher-level properties of the entire network is important, as a stable network may have incredibly high turnover of individual nodes, as evidenced by commercial sex worker networks examined here, where the average time spent in the network is less than 2 years. Despite high turnover and variable network growth over the span of 18 years, we found consistent support that more modular networks tended to stay modular, suggesting that modularity itself is a conserved property associated with stability. This is supported by previous examinations of ecological (Thébault and Fontaine 2010; Grilli et al. 2016; Gilarranz et al. 2017), social (Sales-Pardo et al. 2007), and technological (Duch and Arenas 2006) networks.

Despite this previous, largely theoretical, support for modularity as a signature of stability across multiple systems (Grilli et al. 2016), support for the idea is far from universal (Maynard et al. 2018). Further, the higher-order stability of sexual contact networks—notable commercial sex work networks—has previously not been examined, perhaps out of the assumption that sparse data and massive turnover of individuals over time would obscure any potential relationship. Sparse data are still a persistent concern, even in our commercial sex work networks, as the popularity of the website (*The Erotic Review*) varied over time, such that no perceivable equilibrial dynamics in terms of network properties was ever reached. Related, sex work is currently illegal in the USA, suggesting such that sex workers may change their accounts often. However, these data represent the best temporal data available on sexual contact networks, to our knowledge, and sex workers are unlikely to change their username, as it would cause a loss of their existing client base. We find evidence for this in that numerous sex workers are active in these networks for the

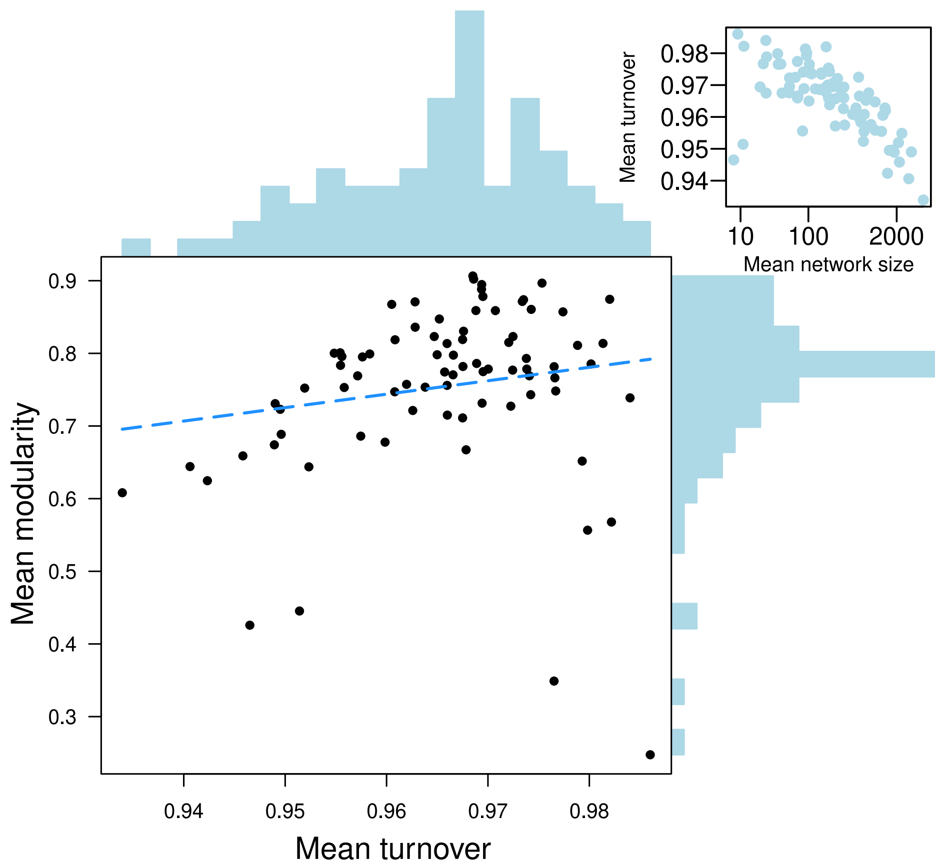

Fig. 4 Mean turnover was not related to mean modularity, whereas it was expected that networks with lower turnover would have higher modularity (e.g., a negative relationship). A dashed line is provided to indicate pattern, but this relationship is not significant. However, we do find a relationship between turnover and network size, indicating that larger networks tend to have less turnover (see inset plot)

entirety of the study period (18 years). A final concern is that higher-order properties of networks (e.g., modularity) are intrinsically related to lower-order network properties (e.g., connectance). We attempted to disentangle the different scales through the use of standardized modularity and partial correlation coefficients considering the relationships between network size, connectance, and modularity, but this specter may still influence efforts to establish a link between network structure and network dynamics.

We found that commercial sex work networks that are more modular tend to conserve this modularity. But what makes a commercial sex work network modular, and how does this relationship scale to other network types? While many systems can be represented as complex networks, the links describing the flow of information or interaction between nodes change in identity and currency. If modularity is signature of network stability, the scale at which this can be generalized is important, as there may be no other *a priori* reason to think that the stock market (Heiberger 2014), the world wide web (Eriksen et al. 2003), spatial networks (Gilarranz et al. 2017), protein networks (Maslov and Sneppen 2002), and commercial sex work networks are comparable in terms of network attributes that confer stability. However, it is striking that networks which have such high temporal turnover still maintain modular structure. This suggests that sex workers and clients in these dynamic networks are playing similar functional roles, and that signatures of stability can be found even in incredibly dynamic evolving networks.

Commercial sex work networks are most likely not a proxy for sexual contact networks, but they may (and likely do) have similar signatures of temporal conservation of modularity. Network stability has been explore across different subfields, leading to multiple bodies of research which sometimes are not synthesized together. By exploring these scaling patterns across multiple networks of a single type (commercial sex work networks), we posit that these dynamical signatures could exist in other temporal networks. A synthesis of which network structures may be signatures of temporal stability, and an understanding of which networks these stability relationships do not hold, will lead to a greater understanding of evolving networks. This understanding has clear implications to the design of sensor and computer networks (Wellman 2001; Bhushan et al. 2008), information spreading in social networks (Liu and Zhang 2014), and epidemic processes in contact networks (Yang et al. 2015).

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Author Contributions TAD performed the analysis. All authors contributed to manuscript writing.

Data Availability *R* code and anonymized data are available on figshare at https://doi.org/10.6084/m9.figshare.14186663.v1.

Declarations

Conflict of interest The authors have no conflicts of interest to declare.

References

Azevedo RB, Leroi AM (2001) A power law for cells. Proc Natl Acad Sci 98(10):5699–5704

Backstrom L, Huttenlocher D, Kleinberg J, Lan X (2006) Group formation in large social networks: membership, growth, and evolution. In: Proceedings of the 12th ACM SIGKDD international conference on Knowledge discovery and data mining, pp 44–54

Bhushan M, Narasimhan S, Rengaswamy R (2008) Robust sensor network design for fault diagnosis. Comput Chem Eng 32(4–5):1067–1084

Cai Q, Xu HC, Zhou WX (2016) Taylor's law of temporal fluctuation scaling in stock illiquidity. Fluct Noise Lett 15(04):1650029

Duch J, Arenas A (2006) Scaling of fluctuations in traffic on complex networks. Phys Rev Lett 96(21):218702

Eriksen KA, Simonsen I, Maslov S, Sneppen K (2003) Modularity and extreme edges of the Internet. Phys Rev Lett 90(14):148701

Fortuna MA, Stouffer DB, Olesen JM, Jordano P, Mouillot D, Krasnov BR et al (2010) Nestedness versus modularity in ecological networks: two sides of the same coin? J Anim Ecol 79(4):811–817

Gilarranz LJ, Rayfield B, Liñán-Cembrano G, Bascompte J, Gonzalez A (2017) Effects of network modularity on the spread of perturbation impact in experimental metapopulations. Science 357(6347):199–201

Grilli J, Rogers T, Allesina S (2016) Modularity and stability in ecological communities. Nat Commun 7(1):1–10

Heiberger RH (2014) Stock network stability in times of crisis. Phys A 393:376–381

Hsieh CS, Kovářík J, Logan T (2014) How central are clients in sexual networks created by commercial sex? Sci Rep 4:7540

Liljeros F, Edling CR, Amaral LAN, Stanley HE, Åberg Y (2001) The web of human sexual contacts. Nature 411(6840):907–908

Liu C, Zhang ZK (2014) Information spreading on dynamic social networks. Commun Nonlinear Sci Numer Simul 19(4):896–904

Maslov S, Sneppen K (2002) Specificity and stability in topology of protein networks. Science 296(5569):910–913

Maynard DS, Serván CA, Allesina S (2018) Network spandrels reflect ecological assembly. Ecol Lett 21(3):324–334

Meyers LA, Newman M, Pourbohloul B (2006) Predicting epidemics on directed contact networks. J Theor Biol 240(3):400–418

Nadini M, Sun K, Ubaldi E, Starnini M, Rizzo A, Perra N (2018) Epidemic spreading in modular time-varying networks. Sci Rep 8(1):1–11

Ndeffo Mbah ML, Liu J, Bauch CT, Tekel YI, Medlock J, Meyers LA et al (2012) The impact of imitation on vaccination behavior in social contact networks. PLoS Comput Biol 8(4):e1002469

Newman ME (2012) Communities, modules and large-scale structure in networks. Nat Phys 8(1):25–31

Poisot T, Gravel D (2014) When is an ecological network complex? Connectance drives degree distribution and emerging network properties. PeerJ 2:e251

Pons P, Latapy M (2005)Computing communities in large networks using random walks. In: International symposium on computer and information sciences. Springer, pp 284–293

Reuman DC, Zhao L, Sheppard LW, Reid PC, Cohen JE (2017) Synchrony affects Taylor's law in theory and data. Proc Natl Acad Sci 114(26):6788–6793

Robinson K, Cohen T, Colijn C (2012) The dynamics of sexual contact networks: effects on disease spread and control. Theor Popul Biol 81(2):89–96

Rocha LE, Liljeros F, Holme P (2010) Information dynamics shape the sexual networks of Internet-mediated prostitution. Proc Natl Acad Sci 107(13):5706–5711

Sah P, Leu ST, Cross PC, Hudson PJ, Bansal S (2017) Unraveling the disease consequences and mechanisms of modular structure in animal social networks. Proc Natl Acad Sci 114(16):4165–4170

Sales-Pardo M, Guimera R, Moreira AA, Amaral LAN (2007) Extracting the hierarchical organization of complex systems. Proc Natl Acad Sci 104(39):15224–15229

Song C, Saavedra S (2020) Telling ecological networks apart by their structure: an environment-dependent approach. PLoS Comput Biol 16(4):e1007787

Staniczenko PP, Kopp JC, Allesina S (2013) The ghost of nestedness in ecological networks. Nat Commun 4(1):1–6

Thébault E, Fontaine C (2010) Stability of ecological communities and the architecture of mutualistic and trophic networks. Science 329(5993):853–856

Tylianakis JM, Morris RJ (2017) Ecological networks across environmental gradients. Annu Rev Ecol Evol Syst 48:25–48

Wellman B (2001) Computer networks as social networks. Science 293(5537):2031–2034

Xiao X, Locey KJ, White EP (2015) A process-independent explanation for the general form of Taylor's law. Am Nat 186(2):E51–E60

Xie M, Jia Z, Chen Y, Deng Q (2012) Simulating the spreading of two competing public opinion information on complex network. Appl Math 3(9):1074–1078

Yang H, Tang M, Gross T (2015) Large epidemic thresholds emerge in heterogeneous networks of heterogeneous nodes. Sci Rep 5(1):1–12

Yoon S, Goltsev A, Mendes J (2018) Structural stability of interaction networks against negative external fields. Phys Rev E 97(4):042311

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